Laboratory Manual for ELEMENTARY STATISTICAL METHODS

As Applied to Business and Economic Data

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PREFACE

This is a revised edition of Neiswanger and Haworth, *Laboratory Manual for Elementary Statistical Methods as Applied to Business and Economic Data*, the first edition of which was published in 1942. Dr. William L. Leavitt, Manager, Commercial Research Division, Standard Oil Company (Indiana), who aided in the preparation of the earlier edition when he was a graduate student at Illinois, has joined as a full collaborator in this revised work.

We continue to think the first course in statistical methods in the applied field of economics and business should emphasize interpretation along with the techniques of arriving at descriptive values. The emphasis on analysis and interpretation of results is doubtless the outstanding feature of this Manual. As before, misuses and misinterpretations of statistical data are stressed, and we retain in this edition some problems which are simple exercises in logic in a quantitative setting.

In order to obtain realistic problem situations more data have been introduced here and there than can be used for calculations by students who lack laboratory facilities. In some of these cases, a portion of the work has been set up in tabular form and some calculations completed. There are, however, always a substantial number of computations in each step of the problem to be performed by the student.

Although an elementary course in statistics cannot produce finished technicians, a student who has "worked through" the tasks assigned in this manual should be acquainted with the elementary vocabulary of the language of size, know the leading sources of data, and understand the more commonly used statistical concepts and scientific methods, their uses and misuses, and their application to the solution of business and economic problems.

This Manual, like its predecessors, has been especially prepared for use with the text, *Elementary Statistical Methods as Applied to Business and Economic Data* by William A. Neiswanger. These materials parallel topical arrangements in that text.

Urbana, Illinois

William A. Neiswanger
Floyd B. Haworth
William L. Leavitt

CONTENTS

SECTION I

The Interpretation of Statistics

1. The Nature of Statistical Results (Chapter II of text) Problems 1, 2, 3, 4 and 5
2. The Interpretation of Statistical Data (Chapter II of text) Problems 6, 7, 8, 9, 10, 11, 13, 14 and 15
3. The Calculation and Use of Percentages (Chapter II of text) Problem 16
4. Spurious Accuracy and Rounding of Numbers (Chapter II of text) Problems 12 and 17
5. Summary and Review, Problem 18

THE INTERPRETATION OF STATISTICS

The Nature of Statistical Data

PROBLEM 1. THE USES OF STATISTICS

Determine the truth or falsity of the following statements and discuss briefly:

1. Statistical methods can be applied equally well to either individual cases or to group phenomena.
2. The use of statistical methods will relieve the business man of the necessity of relying on his own judgment in meeting business problems.
3. Bias in statistical data can result only from dishonesty on the part of the person collecting the information.

PROBLEM 2. FROM GROUP DATA TO INDIVIDUAL CASES

TABLE 1

AVERAGE FUTURE LIFETIME AT GIVEN AGES
United States, 1939–1941

Age	*Future Lifetime in Years*	
	Male	*Female*
0	62.8	67.3
10	57.0	60.1
20	47.8	51.4
30	38.8	42.2
40	30.0	33.2
50	22.0	24.7
60	15.0	17.0
70	9.4	10.5
80	5.4	5.9
90	3.1	3.2
100	1.9	1.9

Source: Based on 1940 Census of Population and Deaths of 1939–41, *U. S. Life Tables and Actuarial Tables*, U. S. Public Health Service, National Office of Vital Statistics, Federal Security Agency.

According to the data, Table 1, 20-year-old males have an average future lifetime of 47.8 years. For 20-year-old females the figure is 51.4 years. Does this mean that:

1. An individual female selected at random from your statistics class will live longer than an individual male student of the same age?
2. Husbands should expect their wives, of approximately equal age, to outlive them?
3. A life insurance company would not be interested in such data as these because insurance is usually sold to individual persons whose life time no statistic can foretell?
4. These data are of no practical value because they cannot be applied to the individual case?

PROBLEM 3. FROM GROUP DATA TO INDIVIDUAL CASES

A school of education has suggested the following standard for the distribution of final grades for students:

TABLE 2

STANDARD PERCENTAGE DISTRIBUTION OF GRADES

Grade	*Numerical Equivalent*	*Per Cent of Students Receiving Grade*
A	93–100	10
B	85– 92	20
C	77– 84	35
D	70– 76	20
E	60– 69	10
F	Below 60	5
		100

Questions

1. Might this distribution be applied to the final grades in a course containing 300 students? Why?

2. Should this distribution be applied to each of fifteen sections making up a course of 300 students? State your reasons.

PROBLEM 4. THE INERTIA OF LARGE NUMBERS

Table 3 shows the yield per acre of corn in the state of Iowa and in the entire United States, 1930–1947.

TABLE 3

YIELD OF CORN PER ACRE IN THE STATE OF IOWA AND THE UNITED STATES
1930–1947

Year	*A* (*Bushels*)	*B* (*Bushels*)
1930	34.0	20.5
1931	32.9	24.1
1932	43.0	26.5
1933	39.5	22.6
1934	23.0	15.7
1935	38.0	24.0
1936	20.7	16.2
1937	45.0	28.1
1938	46.0	27.7
1939	52.0	29.2
1940	52.5	28.4
1941	51.0	31.1
1942	60.0	25.1
1943	56.5	32.2
1944	52.5	32.8
1945	44.5	32.7
1946	57.0	36.7
1947	30.5	28.6

Source: *Agricultural Statistics*, 1948, p. 43, and *Crops and Markets*, December issue 1932–1946, 1947 to 1949 Annual Report, U. S. Department of Agriculture.

PROBLEM 4. (Continued)

Questions

1. Which column represents Iowa experience and which applies to the United States? How do you know?
2. Note that yields are higher in recent years than earlier years. How do you account for this?

PROBLEM 5. THE INERTIA OF LARGE NUMBERS

Among the thousands of students who enroll in the Urbana divisions of the University of Illinois each fall, there are always a few who find it desirable or necessary to withdraw after paying their fees and starting the work of the semester. Table 4 contains the record of enrollments, withdrawals, and the percentage of withdrawals during the prewar and postwar years.

TABLE 4

TOTAL ENROLLMENT, WITHDRAWALS, AND PERCENTAGE OF WITHDRAWALS, UNIVERSITY OF ILLINOIS, URBANA DIVISIONS, FALL SEMESTERS, 1937–1940 AND 1947–1949

Date	*Number Enrolled*	*Number Withdrawn*	*Per Cent Withdrawn*
1937, fall semester	12,571	428	3.40
1938, fall semester	12,816	373	2.91
1939, fall semester	12,389	377	3.04
1940, fall semester	12,524	437	3.49
1947, fall semester	19,477	547	2.81
1948, fall semester	19,166	496	2.59
1949, fall semester	19,675	521	2.64

Source: *Annual Report of the Director*, Office of Admissions and Records, University of Illinois, Urbana, Ill.

Questions

1. Do you think it possible for the Registrar to predict who, among the many thousand students, will withdraw from the University? Why?
2. Do you think it possible for the Registrar to predict with reasonable accuracy how many students will withdraw from the University during a peacetime fall semester? Why?
3. How do you explain the reduction in the per cent withdrawn between the prewar and postwar periods? Do you think that the lower withdrawal rate will continue?

TABLE 5

CREDIT RATINGS OF INDIVIDUALS IN VARIOUS OCCUPATIONS,* UNITED STATES, 1941

Occupation	*Ratings in 1941†*		
	Average Rating	*Ratings by Credit Bureaus*	*Ratings by Stores*
Business executives	95.3	97.1	93.5
Army officers	92.2	90.2	94.1
Chain store managers	92.1	93.4	90.7
Skilled factory workers	89.5	90.4	88.5
Office workers	89.1	91.0	87.1
Trainmen	89.0	90.0	88.0
Retailers (independent)	88.6	91.0	86.1
Civil Service employees	87.5	....	87.5
Engineers (civil, etc.)	87.4	83.0	91.8
Farmers (owners)	84.7	84.2	85.1
School teachers	83.5	85.8	81.2
Doctors	83.2	88.5	77.8
Retail salespeople	81.7	85.6	77.7
Dentists	81.5	87.2	75.8
Postal employees	77.6	82.9	72.2
Nurses	77.3	80.0	74.5
Traveling salesmen (wholesale)	74.3	81.4	67.1
Ministers	73.4	75.6	71.1
Contractors	68.3	70.8	65.8
Policemen	63.0	64.3	61.7
Lawyers	61.0	62.7	59.2
Railroad section hands	60.3	59.4	61.2
College students	59.5	60.0	59.0
Plumbers	59.3	....	59.3
Carpenters	57.4	58.5	56.2
Unskilled factory workers	57.2	57.9	56.4
Janitors	54.6	58.7	50.5
Farmers (tenants)	49.8	48.1	51.4
Common laborers	47.8	45.2	50.4
Domestic servants	47.1	47.5	46.7
Coal miners	46.9	46.0	47.7
Waiters (hotel, restaurant, etc.)	46.7	53.3	40.0
Barbers	45.7	47.1	44.3
Plasterers	45.7	47.6	43.8
Soldiers (enlisted men)	44.6	44.4	44.8
Bartenders	42.3	45.6	39.0
Painters	36.1	33.3	38.8
Farm laborers	35.8	36.6	35.0
Musicians	33.5	33.3	33.6

* Percentage ratings were determined according to the following scale: good, 100; fair, 60; poor, 20.

† Each of the 39 ratings listed in the table for 1941 is based upon 162 to 1,280 reports. The following six occupations were rated by less than 100 but more than 25 credit executives: architects, 72.9; firemen (city), 62.8; oil field workers, 61.4; miners (other than coal), 49.1; lumberjacks, 41.5; truckers, 34.6.

Source: P. D. Converse, "The Occupational Credit Pattern," *Opinion and Comment*, Bureau of Business Research, University of Illinois, Urbana, Illinois, August 12, 1941, p. 2.

The ratings in Table 5 resulted from a study made by the Associated Credit Bureaus of America, the Credit Management Division of the National Retail Dry Goods Association, and the Bureau of Business Research of the University of Illinois. Reports were received from 512 credit bureaus and 819 stores distributed widely throughout the United States.

PROBLEM 6. (Continued)

Questions

1. May we conclude that an individual whose occupation is listed in the lower half of the table will prove to be a poorer credit risk than one whose occupation is in the upper half of the table? Explain.

2. In spite of these results, "some credit men say that occupation is of little importance in passing upon an application for credit" (from page 1, *ibid.*). How can this position be maintained in view of the above results?

3. In view of your answer to questions 1 and 2, how precisely should the ratings shown in Table 5 be interpreted by the credit manager? Explain in detail.

PROBLEM 7. REASONING TO A STATISTICAL GENERALIZATION

February 24, 1950, the General Motors Corporation announced a reduction in wages and car prices under its flexible cost of living wage contract with the United Automobile Workers, CIO. Wages of some 290,000 hourly paid and 72,000 salaried employees were cut approximately two cents an hour and passenger car and truck prices were lowered from $10 to $40 depending on the make and model.

Under the wage contract with UAW wages may be adjusted quarterly to changes in the Consumer Price Index published by the Bureau of Labor Statistics, Department of Labor. This feature of the wage contract is known as an "escalation clause."

During the immediately preceding quarter also, wages and car prices were reduced when the index fell.

Question

May we generalize from this that there is a high correlation between hourly wage rates in industry and commodity prices?

PROBLEM 8. REASONING TO A STATISTICAL GENERALIZATION

Between 1939 and 1948, the cash receipts from farming in the United States rose from $7.8 billion a year to the large value, $31.0 billion. During the same period, the gross national product of the American economy increased from $91.3 billion to $262.4 billion and the country was in a state of great prosperity.

Source: *Survey of Current Business*, Statistical Supplement, 1949.

Question

Do you think these relations can be taken to prove the often repeated phrase, "Prosperity on the farm means prosperity for all"?

PROBLEM 9. REASONING TO A STATISTICAL GENERALIZATION

"Contrary to what any taxpayer might assume at first thought, tax exemption for public housing saves money for taxpayers. The payment in both 1944 and 1945 was $85,453, which is exactly 251 per cent more than the taxes formerly assessed (but never collected in full) on the sites of the two projects."

Source: *Old Brick* Annual Report, July 1946, St. Louis Housing Authority.

PROBLEM 9. (Continued)

Questions

1. What is your estimate of the amount of taxes formerly assessed against the sites on which the public housing was placed?
2. Does the argument made convince you that the taxpayers' first thoughts are wrong? Explain.

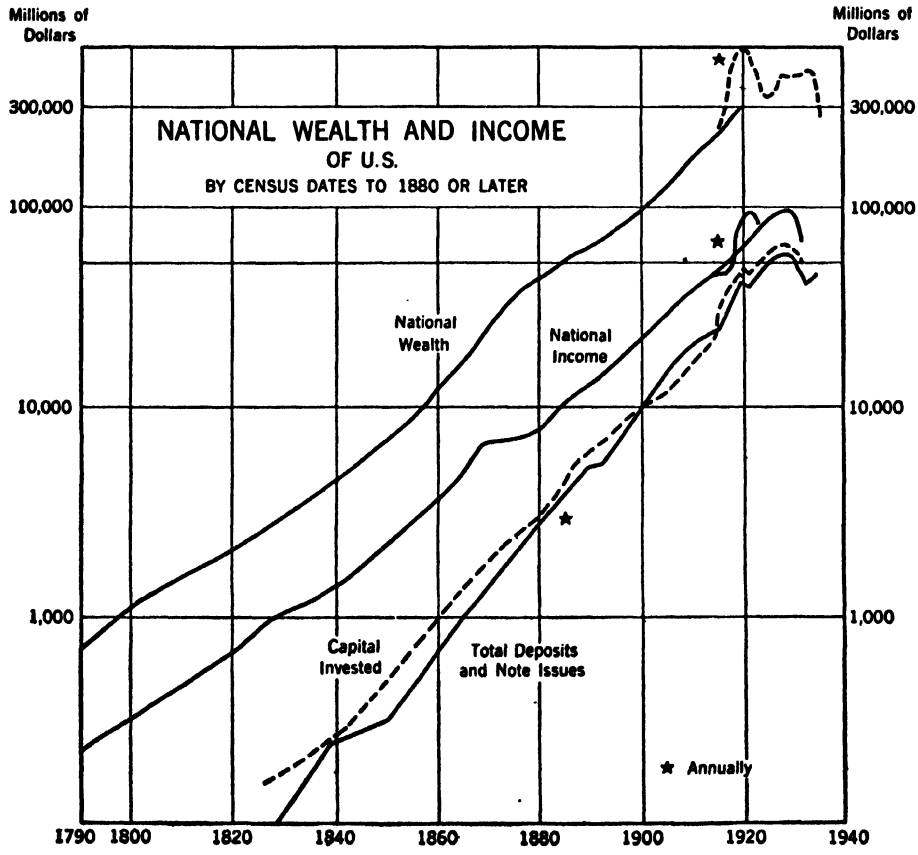
PROBLEM 10. CONCURRENCE IN TIME—CAUSATION?

A book, *Capitalism the Creator*, by Mr. Carl Snyder was published in 1940. Mr. Snyder's thesis is given, briefly, in the following quotation:

Who, and what, created this industry—so incredibly vast? Labor? In a sense, "labor" contributed almost nothing. It did not invent the dynamo, which produces all this electric power. That was the work of an untutored bookbinder's apprentice, who lives in fame as the greatest experimental genius of his age, and perhaps since Archimedes. Nor did "labor" make any serious contribution to develop it. To perfect and make practical this epochal invention required the genius and skill of three generations and more, of thousands of trained engineers and physicists, a brilliant line extending from Faraday to Edison, Steinmetz and Langmuir. Who paid for all this long travail of experimentation, and the salaries of all these technicians, involving the investment of tens of millions of dollars? Labor? No, it was Capital Savings; this, and this alone which alike created this wondrous industry, and all our modern world of comfort, convenience, and luxury beside.

Such is the thesis of this volume. It is not offered as a "theory," but as an elemental reality. . . . The thesis here presented is simple, and unequivocal; in its general outline, not new. What is new, I would fain believe, is the proof; clear, statistical, and factual evidence. . . .

Part of the statistical proof offered is the following chart and comment:



PROBLEM 10. (Continued)

We have official estimates of the increase of National Wealth, that is, taxable wealth, from 1850, and back of that some careful estimates by the English statistician Mulhall. Professor W. I. King has made similar estimates of National Income, from 1850; these have been here carried back to 1790 by comparison with the increase in production and trade. With these are shown capital invested in manufacturing, and total deposits in banks, revealing how closely all of these factors are associated. It is precisely this increase in available capital and bank deposits which has made possible the extraordinary growth of our national income and well-being. And when this capital supply is cut off, we have just such stagnation and social depression as in the ten years from 1930—leading to every conceivable economic vagary, and to a colossal loss in wealth, employment, and income. If we could but learn that here, at least, money (i.e., Capital) makes the mare go!

Source: Carl Snyder, *Capitalism the Creator*, The Macmillan Company, New York, 1940, pp. 4 and 89.

Question

1. Without considering the merit of the entire argument which Mr. Snyder develops in his book, consider only the one question: Do the statistics charted *prove* that the increase in capital invested, deposits, and note issues caused the increase in national wealth and income? Explain.

PROBLEM 11. ARE OTHER THINGS EQUAL?

The following comes from the February 6, 1938, issue of the *Chicago Tribune:*

WOMEN CALLED SAFEST DRIVERS OF AUTOMOBILES

Des Moines, Ia., Feb. 5—(AP)—Police Captain F. E. Timmons of the Des Moines traffic detail declared today that women are the safest drivers and he endorsed back seat driving. Speaking on a police sponsored safety broadcast, Captain Timmons declared that while women motorists held twenty-five per cent of the drivers' licenses issued in Iowa, they figure in only nine per cent of the accidents reported to police and state motor vehicle department. . . .

Question

1. Do these figures prove that women drive more safely than men?

PROBLEM 12. STATISTICAL ESTIMATES AND SPURIOUS ACCURACY

1. The U. S. Treasury Department reports the gross debt per capita in 1948 as $1,721.29. A footnote states that this value is based on estimated population for the continental United States as of July 1.

Source: *Annual Report of the Secretary of the Treasury* on the State of the Finances; for the Fiscal Year ended June 30 1948, p. 445. Treasury Department, Washington, D. C.

2. Average hourly earnings for production workers in manufacturing industries, United States, August 1949, are: gross $1.399, excluding overtime, $1.366.

Source: *Monthly Labor Review*, January, 1950, p. 111. Bureau of Labor Statistics, Department of Labor. Washington, D. C.

3. The population of Chicago at the time of the last census of population (1950) was 3,631,835.

Source: *17th Decennial Census—1950*, Preliminary Population Announcement, Department of Commerce, Bureau of the Census, Chicago, July 1950.

Questions

1. In each of the three cases cited, do you think the precision of the measurements reported is less than the figures indicate? Explain.

2. Would you argue that less precise statements might be better?

PROBLEM 13. QUESTIONS CONCERNING STATISTICAL UNITS

1. How, according to the census definition of a farm given in the text, pp. 39–41, might a change in the price of farm produce cause a piece of land to be classified as a farm in one census and not a farm in another, without any change in the use of the land?

2. The price of gas was increased from $1.10 to $1.30 per 1000 cubic feet between June and December, 1932, in the city of St. Louis, Missouri. What questions would you raise before accepting this statement as a true expression of the change in the cost of this fuel in St. Louis?

3. Personal income in the United States was estimated at 72.6 billion dollars in 1939. Personal income in 1948 was 211.9 billion dollars. What questions would you raise before accepting these figures as evidence that the living standards of the American people were higher in 1948 than in 1939?

PROBLEM 14. QUESTION OF STATISTICAL UNITS

The June 16, 1947, issue of *Life* magazine carried an article in which the Princeton graduating class of 1932 was analyzed and shown to be superior to other Princeton classes with respect to average salary earned by its members 15 years after graduation. For instance, the average earnings of the members of the class of 1921 were only $5,289 by 1936, 15 years after graduation, but members of the class of 1932 were earning an average of $9,543 in 1947 at the end of their first 15 years out of school.

The author explained the superior showing of the 1932 class, "graduating in the depth of the depression sharpened their wits and taught them how to find a job and work hard to hold onto it."

Questions

1. Does the author's explanation seem reasonable to you? Explain.

2. Can you think of a better explanation of the variation in average earnings, class by class, 15 years after graduation? What is it?

PROBLEM 15. THE ASSUMPTION OF STATIC CONDITIONS

In 1939, the production of a dozen men's shirts required 7.06 man-hours of direct and indirect factory labor. By 1947, comparable labor requirements were reduced to 5.96 man-hours.

Source: *Man-Hours Expended Per Unit for the Manufacture of Men's Dress Shirts in Selected Establishments, 1939 to 1947*, p. 1. Report No. LS-48-3232, Bureau of Labor Statistics, U. S. Department of Labor, July 1948.

Question

1. This reduction of 15.6% in man-hour requirements means that about 16% fewer persons may expect to find employment in this industry. Right?

PROBLEM 16. REPORTING AND INTERPRETING PER CENT CHANGES

1. The following is a paragraph from a letter in which a manufacturer is justifying an increase in his price:

We purchase Item J from the Ball and Roller Bearing Company, Danbury, Connecticut, who changed the discount from 40% to 30%. This is a difference of 10% in the discount but is a net increase of 16⅔% to us.

Question

Would you consider this a possible result or has the manufacturer made a mistake?

2. If we have 100 numbers, each with a one per cent error, we will, if we add the numbers together, secure a 100 per cent error in the total. Do you agree?

3. The following quotation is taken from a weekly publication entitled *In Fact.*

PROBLEM 16. (Continued)

The National City Bank, in its August Bulletin, analyzes the profits of leading corporations for the half year ending June 30, 1941. *In Fact* compares these figures with the comparable periods of 1940 and 1939.

	Industrial Groups	*Companies Reporting*	
1941	25	360	increase in profits 1941 over 1940 of 20.3%
1940	30	400	increase in profits 1940 over 1939 of 58.6%

This percentage increase of 78.9 is large enough, but when the bulletin of August, 1939, shows an increase in net profits over 1938 of a cool 100%, the recapitulation is staggering. Simple arithmetic discloses an increase in net profits, 1941 over 1938, of 178.9 per cent.

Source: "More about the Unfair Tax Program," *In Fact*, Sept. 8, 1941, p. 2.

Point out the errors in the statistical analysis.

4. This quotation also comes from *In Fact*.

Here . . . are the facts. (Corporate profits, of course, mean that the public paid more money; wages, it must be remembered, constitute only a small proportion of value of products, 6% in meat, 21% in steel, 16% in autos.)

(Column 1 is from U. S. Department of Labor except aircraft, which is from Department of Commerce; column 2 is from National City Bank letter, March 1941.)

1940 Increases Over 1939

	Weekly Wage Increase	*Corporation Net Profits*
All manufacturing	6%	27%
Aircraft	3%	191%
Iron, steel	5%	98%
Machinery	8%	68%
Transportation (mainly autos)	7%	25%
Textiles	2%	33%

On August 26 the American Federation of Labor issued a statement showing that wage gains in the past year do not warrant the slightest advance in prices and that industry's profits are so immense they can easily absorb the moderate wage increases; furthermore, worker's increasing productivity steadily reduced unit labor costs and expanding industrial production is reducing unit overhead costs.

Source: "Newspapers Blame Farmers, Labor," *In Fact*, September 8, 1941, p. 2.

Question

Do the statistics *prove* the point?

5. A report is received from industry by a government agency containing the following argument:

In melting, however, 25% of the metal is lost. Any pricing formula you impose on the industry must, therefore, provide that the per pound price of the metal paid by the founder be increased 25% in figuring metal cost.

Question

Do you find anything wrong with the arithmetic of this argument?

6. Quote from a political spellbinder during the dark days of the great depression, 1929–1937.

Prices have declined 25 per cent. Do you know how much they will have to rise to get back where they were? Thirty-three and one-third per cent! We'll never make it!

PROBLEM 16. (Continued)

Question

Is his arithmetic right? What of his conclusion?

7. The Educational Policy Committee of a large middle western University states in a recent report:

> . . . small classes are relatively expensive and a large per cent of such classes within a department, where conditions do not require individual instruction or small classes, suggest that the list of offerings might be more wisely planned.

In an attached table a department is shown which enrolls less than ten students in 50 per cent of its classes. There is a strong implication that the course offerings of this department stand in need of reorganization.

Question

What questions would you raise?

8. $583 million of securities were issued during November 1948 for the purpose of raising new capital in the United States. During November 1949 but $379 million were issued for this purpose, according to the *Federal Reserve Bulletin* of January 1950. This represents approximately a 65 per cent decline. Right?

9. During the war a manufacturer explained a price increase as follows:

> We have been hit hard by material allocations. As a matter of cold and brutal fact there has been a 150% decline in our production in the last quarter compared with the same quarter last year. You know what this does to costs.

Question

What common error do you find in this statement?

10. The following index numbers are taken from the *Monthly Labor Review* published by the Bureau of Labor Statistics, Department of Labor.

TABLE 6

INDEX NUMBERS OF WHOLESALE PRICES BY GROUPS AND SUBGROUPS
1926 = 100

Group and Subgroup	*1949*	*1948*	*1941*
All commodities	155.0	165.0	87.3
Farm products	165.5	188.3	82.4
Textile products	140.4	148.6	84.8
Farm machinery	146.5	136.9	94.5
Iron and steel	165.7	155.2	96.4
Nonferrous metals	144.3	157.5	84.4
Lumber	286.0	312.2	122.5
Chemicals	117.4	126.1	87.2
Oils and fats	123.8	205.0	77.6
Paper and pulp	160.8	168.5	98.2

Source: *Monthly Labor Review*, Bureau of Labor Statistics, Department of Labor.

Questions

a. How would you state the increase in lumber prices to 1949? 286.0 per cent increase? 186.0 per cent increase? Or how?

PROBLEM 16. (Continued)

b. Over what period of time has the increase taken place? That is, what is the reference period?

c. How should we write the percentage decrease in chemicals between 1948 and 1949? Is it 8.7 per cent?

d. Which of the commodity groups shown above experienced the largest percentage increase 1941–1948? How should the increase be written?

e. The All Commodity Index is 87.3 for 1941. Can we conclude that prices in 1941 were below "normal?"

f. There are questions concerning sampling and averaging which might well be raised regarding these figures. What are some of these questions?

PROBLEM 17. SPURIOUS ACCURACY

1. Round the following to two decimal places:

(a) 717.8552	(c) 24.9450
(b) 863.5845	(d) 813.8550

2. The University of Michigan had a total attendance of 565,432 (turnstile count) at its first six home football games in 1949. What was the average attendance per game?

3. According to the Federal Reserve Board, $20 bills in circulation totaled $8,520 million in November, 1949. This is 31 per cent of the total money in circulation.

What is your estimate of the total money in circulation?

4. As of October, 1947, all member banks of the Federal Reserve System had $97,328 million in loans and investments. Of this, 60.9 per cent was invested in U. S. government obligations.

What is your estimate of the amount member banks had invested in U. S. government obligations?

5. The Research Department of the Sopac Auto Supply Company has been requested to provide estimates of sales for the next year. Each of the four lines is assigned to an individual analyst and a tabulation of their reported findings is as follows:

	Thousands of Dollars
Battery sales	60
Tire sales	230
Tube sales	32.84
Auto accessories	81.608

Questions

a. What is the total of the above sales estimates?

b. In order to avoid loss of time in the preparation of the forecasts, what instructions should the supervisor have given each analyst relative to the required precision in the estimates?

General Questions Problem 17

1. What rule have you followed in rounding the numbers in group 1 above?

2. What rule have you followed in deciding the number of significant places in the addition in 5 above?

3. What rule have you applied in determining the number of significant places in multiplication and division?

PROBLEM 18. REVIEW OF SECTION I

In Section I many common mistakes made in the use of statistical data have been illustrated. Attention has been called to these faults in reasoning and computations because they *are* common and one must be continually on guard against them.

In order to systematize and organize the points illustrated by Problems 1–17 inclusive, write a brief report enumerating and classifying the types of faulty procedures illustrated in this section.

SECTION II

Sources and the Collection of Data

1. The Road Map of a Survey (Chapter III of text) Problem 19
2. Questionnaire and Schedule Construction (Chapter III of text) Problems 20, 21 and 22
3. Sources of Business Data (Chapter III of text) Problem 23

PROBLEM 19. ROAD MAP OF A SURVEY

The following constitutes a general check list of survey procedures. Although the steps outlined below have been presented as though they were in direct sequence, considerable retracing and adjustment is typically required.

1. *Preparation of a prospectus*
   Tentative statement of the problem, data needed, general estimate of cost and length of time required for completion.
2. *Authorization to perform the study*
   Approval is given to undertake a specific research project along with authorization for expenditure of funds as required within stated limits.
3. *Budgeting the funds authorized for the study*
   An approximation is made of the cost requirements for each phase of the survey, the length of time estimated as necessary to complete the given phase of the study by stenographic, clerical, comptometer, coding, and the various analytical groups.
4. *Defining the purpose of the survey*
   The purpose of the survey is stated with the greatest amount of precision possible—what the survey is intended to accomplish—its use in the hands of management.
5. *Reviewing previous or similar studies*
   The analyst should be familiar with other work done in the same or closely related fields both within the company and by other research groups, in order to be guided by other findings and other experience as well as to avoid unnecessary effort or expense.
6. *Examination of relevant characteristics of the subject to be surveyed*
   The greater the familiarity with pertinent behavior patterns—economic, sociological, ethnic, and the like, as appropriate, or other significant attributes of the subject being surveyed, the greater guidance the analyst has in placing each analytical phase of the study in proper context.
7. *Definition of the universe*
   Statement is required delimiting the scope of the survey as to qualified respondents or area of investigation.
8. *Preparation of questionnaire or schedule*
   a. List questions the survey should answer.
   b. Review with interested parties.
   Ascertain whether answers to questions propounded would serve the purposes of various groups who plan to use the findings of the survey. For example, the Advertising Department, Sales Department, Credit Department.
   c. Draft questionnaire.
   d. Pre-test.
   e. Revise questionnaire or schedule in light of pre-test findings.
   f. Pre-code where possible.
   g. List and outline the form of analytical tables planned.
   Plan the cross classifications of the variables as appropriate to the general and specific purposes of the study.
9. *Design of the sample*
   Determination of type of sample—unrestricted random, quota, area, cluster, and stratifications, if any. The accuracy required and the amount of money available for design and field work are some of the factors which govern the type and size of sample used.
10. *Collection of data*
    a. Training interviewers
    Effort must be made to assure interviewer understanding of the schedule, comprehension of the questions contained in the schedule, effective interviewing procedure and uniformity of approach.

b. Supervising interviewers
Arranging the iterinary, keeping account of the progress made, verifying that interviews were made in accordance with requirements, etc.

c. Conducting the interviews
Eliciting the required information from the proper respondent in accordance with the prescribed plan.

d. Call-backs and substitutions
Some interviews cannot be made because respondents are not at home, for example, or refuse to answer. Call-backs may be required or a system of substitution may be planned which will not introduce bias.

e. Reviewing interviewer experience
As the interviews progress, steps should be taken to assure that the operating experience of the interviewers is conveyed to the analyst both as a guide for interpreting the findings, to evaluate the significance of particular questions in the eyes of the respondent and to record experience on specific aspects of the schedule or data collection procedures.

11. *Receipt and review of returns*
Returns are reviewed for completeness, rectifiable inconsistencies, etc. Where necessary the form is returned to the interviewer for re-interview, as required.

12. *Editing, coding, verifying*
After personal examination of a sufficient number of forms to establish the adequacy and internal consistency of the answers, procedures are established for editing all forms, for classification and coding of the answers obtained and for verifying the clerical coding operations.

13. *Tabulation of results*
Where the survey is large and/or the variables numerous, tabulating equipment may be required. In this event, cards must be punched for the tabulating equipment and the tables run on tabulating machines. Under these circumstances, plans will have been made earlier to assure that the form in which the data are coded and the requirements for tables are appropriate to efficient use of mechanical tabulating procedures.

14. *Review of procedures and results*
Although the analyst must stay alert to any change which would require review throughout the study, a review of the procedures and results at this stage of the study is warranted. Appropriate adjustments are made where required.

15. *Organization of the report*
A tentative outline of the written form of the report is prepared in order to provide a systematic approach to preparation of the report.

16. *Analysis of data and preparation of the report*
Not only must the analysis be of competent quality, but it is important that the results be presented in such a way as to be understood by, and of interest to, management. Usually the findings will contain the basis for recommendations. It is particularly important that these be presented in an interesting fashion with clarity and with the maximum amount of tact.

17. *Completion of the technical review*
Throughout the study the analyst should prepare a running notation of procedures involved and commentary on their effectiveness. This technical review will serve both as a guide to interpretation of the findings by other analysts and as an aid in crystallizing the experience gained in the study.

Schedule and Questionnaire Design

PROBLEM 20. WHAT IS WRONG WITH THIS QUESTIONNAIRE?

In the development of a survey, the prepation of the questionnaire or interviewers' schedule ranks high in both difficulty and importance. The hypothetical questionnaire which appears below has been designed to illustrate some of the more common pitfalls of questionnaire preparation.

Assume that you are a market analyst and that this questionnaire on "Home Preference in Hand Soap" has been submitted for your approval.

PROBLEM 20. (Continued)

Home Preference in Hand Soap

a. Name the brand of hand soap you prefer—such as Palmolive, Ivory, or the like. _____ (Brand)

b. What brand of hand soap did you buy last? _____ (Brand)

c. Why did you buy (*soap named in b*) hand soap? _____

_____

d. Would you prefer a fatty acid or a detergent base soap? _____

_____ (Preference)

e. What brand of hand soap did you use this morning? _____ (Brand)

f. How many cakes of (*soap named in e*) do you use in a year? _____ (Number)

g. What is the total quantity of hand soap your household uses in a year? _____ (Number)

h. Where have you seen or heard the soap named in "*b*" advertised?

1. Soap operas
2. Newspapers
3. Store displays
4. Singing commercials
5. Prize contests
6. Billboards
7. Magazines
8. Other commercials

Discussion

Evaluate each of these questions as either satisfactory or unsatisfactory, indicating the nature of the faults recognized.

PROBLEM 21. SCHEDULE CONSTRUCTION

Sections A, C, and the Classification Data section of a comprehensive schedule prepared and used by Market Facts, Inc., Chicago, in a study of livestock marketing are shown in Exhibits 1 and 2 on the following pages. The personal interview method was employed.

The purpose of the survey was stated to be:

To secure an accurate, up-to-date picture of *how* livestock is marketed today, the importance of various market outlets, and the factors contributing to the use of these outlets by livestock producers.

Questions

1. In what respects is this schedule superior to the one shown in Problem 20? Explain.
2. Would this survey answer the question, "Is the farm trucker a key person in determining the flow of livestock to particular markets?" Explain.
3. What terms in this schedule should be defined for the guidance of the interviewers? Make a list.
4. Would this survey help answer the question, "What influences determine the farmer's choice of market to which to sell his stock?" Explain.
5. How would the classification data be used in the analytical parts of this study? Explain.

EXHIBIT 1

SECTIONS A, C AND THE CLASSIFICATION DATA SECTION, FROM A COMPREHENSIVE SCHEDULE BY MARKET FACTS, INC., 1950

MARKET FACTS, INC., 39 S. LASALLE ST., CHICAGO JOB NO. 9-353 #

NAME AND INITIALS OF PERSON INTERVIEWED _____

POSTAL ADDRESS _____

CITY OR TOWN _____ STATE (4) _____ 4 5

COUNTY _____ (5) TOWNSHIP _____ (6, 7) 6 7

ZONE # _____ (8) SEGMENT # _____ (9, 10) FARM # _____

DATE(s) OF CALL(S): 1. _____ 2. _____ 3. _____

SECTION A -- ASK OF ALL RESPONDENTS

1. At what time of the day do you find it most convenient to listen to the radio? _____ (11) 8

2. What radio stations do you listen to most often? _____ (12) 9 10

3. What farm papers or magazines do you find most helpful? _____ (13) 11 12

4a. Are you farming as a hired manager, in a partnership, or for yourself?

FOR SELF ____ IN PARTNERSHIP ____ AS HIRED MANAGER ____ (14) 13

(If "For self", skip to Question 5)

4b. *(If "Partnership" or "Hired Manager" in Question 4a, ask:)* Who takes part in making the final decisions as far as the sale of your livestock is concerned?

SELF ____ PARTNER ____ OWNER ____ DOES NOT RAISE LIVESTOCK FOR SLAUGHTER ____ (15) 14

(If "Self" mentioned in 4b, skip to Question 5. If "Does not raise livestock for slaughter", skip to Classification Data.)

4c. *(If "Self" not mentioned in 4b, ask:)* Who is the partner/owner?

NAME _____

POSTAL ADDRESS _____ (16) 15

(Continue interview with respondent, but also interview person listed in 4c, if available.)

5. Where do you usually get your price information on livestock? _____ (17) 16

6a. Who helps you decide which selling point you will sell your livestock to? In other words, do you talk or write to anyone in an attempt to find out where would be the best place to sell your livestock?

YES ____ NO ____ (18) 17

6b. *(If "Yes", ask:)* Who do you contact? _____ (19) 18

19

JOB NO. 9-353
PAGE FOUR

SECTION C -- ASK OF ALL RESPONDENTS

20. How many times did you sell slaughter cattle during 1948? _____

DID NOT RAISE SLAUGHTER CATTLE FOR SALE _____ (20) 20

(If cattle not sold for slaughter, skip to Section D)

21. About how many cattle did you sell for slaughter during 1948? _____ (21,22,23) 21 22 23

22a. Which of these types of selling points did you use during 1948 for your cattle? *(Show white card)*

22b. *(If more than one selling point used, ask:)* About how many cattle — last year — did you market at each of those selling points? (24-35)

SELLING POINT	QU. #22a "X"	QU. #22a "Y"	
CENTRAL MARKET, PUBLIC STOCKYARDS OR TERMINAL MARKET CITY: *(Write in)* _____			24
CITY: *(Write in)* _____			25 26 27
LOCAL DEALERS -- LOCAL INDEPENDENT BUYERS -- TRUCKER BUYERS NAME _____ CITY _____			28
NAME _____ CITY _____			29 30 31
LOCAL AUCTION MARKETS			32
COOPERATIVE LIVESTOCK MARKETING ASSOCIATION			33 34 35
PACKERS -- LOCAL PACKER BUYERS -- DIRECT TO PACKER AT CHICAGO -- DIRECT TO NEARBY PACKING PLANT NAME _____ CITY _____			
NAME _____ CITY _____			
SOLD TO OTHER FARMERS			
	TOTAL		

Continued on the next page. White and yellow cards referred to in Schedule are shown as Exhibit 2.

EXHIBIT 1 (continued)

Sections A, C and the Classification Data Section, from a Comprehensive Schedule by Market Facts, Inc., 1950

JOB NO. 9-353
PAGE FIVE

23. Which of these *(Show yellow card)* was most important to you when you sold your cattle to _____ *(Question #22a or #22b)*

K. "I usually get more net proceeds for my cattle when I sell them there" _____

L. "It is more convenient for me to sell by cattle there" _____

M. Any other reasons *(Specify)* _____

_____ (36)

36 [ ]

24. *(If "K", ask:)* Why did you feel you would get greater net proceeds there?

_____

_____ (37.38)

25. *(If "L", ask:)* Why did you feel it was more convenient to sell there?

_____

_____ (37.38)

37 38 [ ][ ]

26. *(If "Other" reasons, ask:)* Why is that so important to you?

_____

_____ (37.38)

27. Did you last year ship your cattle in your own truck, did you hire a trucker or, did the buyer pick up your cattle?

USED OWN TRUCK___ HIRED TRUCKER___ BUYER PICKED UP___ (39)

(If "Hired trucker" not mentioned, skip to Section D)

39 [ ]

28. Did the trucker at any time last year suggest where you should sell your cattle?

YES_____ NO_____ DON'T REMEMBER_____ (40)

(If "No" or "Don't remember", skip to Section C)

40 [ ]

29a. Which selling point did he suggest?_____ (41)

41 [ ]

29b. *(If a central market, ask:)* Did he suggest the central market itself, or did he have a commission firm or agency at the central market in mind?

CENTRAL MARKET_____ COMMISSION FIRM_____ (42.43)

42 43 [ ][ ]

30. What reasons did he give for suggesting this selling point?

_____

_____ (42.43)

PAGE SEVEN

31. Did his suggestions influence you in any way as to which selling point you sold your cattle?

YES_____ NO_____ DON'T REMEMBER_____ (44)

44 [ ]

32. The last time a trucker picked up your cattle, did he buy or ask to buy your cattle?

BOUGHT_____ ASKED TO BUY -- BUT DIDN'T_____

DID NOT ASK_____ (45)

45 [ ]

CLASSIFICATION DATA

Approximate age of respondents: 29 OR UNDER___ ; 30-39___ ; 40-49___ ; 50 OR OVER___ (57)

57 58 [ ][ ]

Number of years engaged in farming_____ (58)

Total acres on farm_____ (59-62)

59 60 61 62 [ ][ ][ ][ ]

School last attended: ELEMENTARY___ HIGH SCHOOL___ COLLEGE___ (63)

Operatorship: FULL OWNER_____ ; PART OWNER-PART TENANT_____ ; TENANT_____ ; MANAGER_____ (64)

63 64 [ ][ ]

65 [ ]

Owns truck: YES___ NO___ *(If "Yes")* SIZE___ MAKE_____ (65)

Major source of farm income: LIVESTOCK_____ DAIRY PRODUCTS_____ POULTRY___ FIELD CROPS___ OTHER *(Specify)*_____ (66)

66 [ ]

Distance of farm from Chicago: _____ miles (67-69)

67 68 69 [ ][ ][ ]

Distance of farm from closest central market: _____ miles (70-72)

70 71 72 [ ][ ][ ]

Specify central market *(If other than Chicago)*: _____ (73)

73 [ ]

Market Facts Representative _____ (74,75)

74 75 [ ][ ]

Comments on Interview: _____

_____

_____

_____

_____

76 77 [ ][ ]

78 79 [ ][ ]

REPRESENTATIVE: Do not write below this line

80 [ ]

COOPERATIVE___ LOCAL INDEPENDENT BUYER___ AUCTION___ LOCAL PACKER_____ (80)

White and yellow cards referred to in Schedule are shown as Exhibit 2.

PROBLEM 21. (Continued)

EXHIBIT 2

THE WHITE CARD

Local Auction Markets
Cooperative Livestock Marketing Association
Packers—Local Packer Buyers—Direct to Packer at Chicago—Direct to Nearby Packing Plant
Central Market, Public Stockyards or Terminal Market
Local Dealers—Local Independent Buyers—Trucker Buyers
Sold to Other Farmers

Ques. 9a and 22a

THE YELLOW CARD

K. "I usually get more net proceeds for my cattle when I sell them there"

L. "It is more convenient for me to sell my cattle there"

M. Any other reasons

Ques. 23

Source: Exhibits 1 and 2, from Market Facts, Inc., Mr. W. F. O'Dell, President, Chicago.

PROBLEM 22. UNIVERSITY STUDENTS AND THE FOUNTAIN PEN MARKET

Assume that you are employed by a fountain pen manufacturer who considers the student market important.

Your task is first, to discover what kind of fountain pen students seem to prefer, and second, to find out where students get their pens.

PROBLEM 22. (Continued)

Questions

1. Do you consider the statements of purpose given above sufficiently precise? If not, sharpen them until they are useful. (See point 4 of the Road Map.)
2. Do you think the objectives stated are so different as to require two separate surveys or could both objectives be accomplished in one? Explain.
3. Assume that you are to make one or both of the studies.
   a. Define the universe (point 7 of Road Map).
   b. List the questions your survey is to answer (point 8 of Road Map).
   c. Draft the schedule (questionnaire) which is to be filled in by the personal interview method.
   d. If there are any terms used in your schedule which should be defined to avoid mistakes by your interviewers or respondents, define them.
4. Turn to the Road Map and note what additional steps must be taken after the schedules are received from the field staff.

Sources of Business Data

PROBLEM 23. LIBRARY ASSIGNMENT

Each student may be assigned the task of obtaining one of the series of data from the list below. These data are available in the *Statistical Supplement of the Survey of Current Business*, published by the Department of Commerce, United States Government.

Instructions

1. Go to the *Statistical Supplement of the Survey of Current Business* for the most recent year and copy the annual values for the last 10 years of the series assigned you.
2. Look up and copy the footnotes (there are some 100 pages of footnotes at the back of the *Statistical Supplement*) which relate to your series. If they are very lengthy, abridge them.
3. Check through the *Statistical Supplement* and list one other series of data which you think might aid in explaining the variations observed in the data collected under 1 above. Write a brief paragraph explaining why you selected the one chosen.

Questions

1. What is the primary source of the data you collected from the *Statistical Supplement* under Instruction 1 above?
2. Have the data you collected been "adjusted"? If so, for what and how do they differ in their behavior from "unadjusted" series?
3. Are any terms defined in the footnotes referred to under Instruction 2 above? If yes, please list the terms with the definitions given.
4. Do you think the series you have copied is strictly comparable throughout the period covered? Would any lack of comparability detected serve to bias the figures? Do you have any estimate of the amount of bias, if any?

Partial List of Series to Be Found in

Statistical Supplement, Survey of Current Business

1. Monthly average investment in new plant and equipment by U. S. industries.
2. Total U. S. National Income.
3. Wages and salaries paid to non-governmental employees, U. S.
4. Index of Industrial Production, U. S. Adjusted.
5. Index of Industrial Production, U. S. Unadjusted.

PROBLEM 23. (Continued)

6. Total profits of U. S. corporations before taxes.
7. Expenditures by consumers for non-durable goods, U. S.
8. Total amount spent by consumers for goods and services in the U. S.
9. The value of new construction in the U. S. each year.
10. Total amount of personal savings by individuals in the U. S. each year.
11. Index of iron and steel production.
12. Industrial and commercial failures, compiled by Dun and Bradstreet.
13. Consumers' Price Index, compiled by the Bureau of Labor Statistics.
14. Prices received by U. S. farmers, computed by the Bureau of Agricultural Economics.
15. Prices paid by U. S. farmers, computed by the Bureau of Agricultural Economics.
16. The general parity ratio for farming, computed from price indexes.
17. The total number of businesses operating in the U. S., monthly average.
18. Index of manufacturers' sales adjusted for number of working days per month.
19. Inventories held by retail establishments in the U. S.
20. Index of the price of bituminous coal.
21. Index of wholesale prices, all commodities.
22. The purchasing power of the U. S. dollar as measured by the Consumers' Price Index.
23. Number of production workers engaged in the manufacture of aircraft and parts (exclusive of engines) in the U. S.
24. Number of new dwelling units (non-farm) started each month in the U. S.
25. Cost of radio facilities used in advertising automobiles, clothing, electrical equipment, financial services, foods and beverages, motor fuel, household goods, soap, smoking materials, toilet goods and medical supplies, and the total of all of these for a recent month.
26. Ratio of collections to accounts receivable in U. S. department stores by months.
27. Monthly sales by Sears, Roebuck and Company.
28. Estimated number of people, fourteen years of age and over, in the U. S. civilian labor force, since 1940.
29. New public utility construction in the United States.
30. New private residential construction in the United States.
31. Nation's fire losses, compiled by the National Board of Fire Underwriters.
32. Man-days idle due to labor-management disputes and work stoppages.
33. Admitted assets of the member companies of the Life Insurance Association of America in the form of United States Government bonds and stocks (book value).
34. The value of ordinary life insurance written in New England each month.
35. Bond yields as computed by Moody's Investors Service by ratings; that is, Aaa, Aa, A, Baa.
36. New York Stock Exchange market value of all listed shares.
37. Dow-Jones and Company, Incorporated, average of 30 industrial stock prices.
38. Exports and re-exports of the United States to Africa.
39. Imports to the United States from South American countries.
40. Revenue passengers carried by scheduled airlines.
41. Consumption of tax paid cigarettes.
42. Monthly average production of pig iron reported by American Iron and Steel Institute.
43. Total production of all types of motor fuel.
44. New passenger car registrations compiled by R. L. Polk and Company.
45. Book value—End of month inventories of manufacturers' goods in process, at book value.
46. The Consumers' Price Index prepared by U. S. Department of Labor.
47. Construction Cost Index, San Francisco, prepared by the American Appraisal Company.
48. Sales of automotive parts and accessories by retail stores.
49. Index of department store sales in the Federal Reserve District of Kansas City.
50. Average weekly earnings of employees in the automobile industry, U. S.
51. The discount rate charged by the Federal Reserve Bank of New York.
52. Gross direct public debt of the Federal government outstanding each month.
53. Imports of gold from foreign countries.
54. Index of freight car loadings on Class 1 steam railways.
55. Price of gasoline at retail service stations, 50 cities and at the Oklahoma refinery.
56. Index of construction costs, frame residences, in the U. S.

SECTION III

Simple Random Sampling and the Concept of Error

1. Non-sampling Errors in Sampling (Chapter IV of text) Problem 24, parts 1–4
2. Mistakes, Errors and Bias (Chapter IV of text) Problems 25, 26, 27
3. Selecting a Sample (Chapter IV of text) Problems 28, 29, 30 and 31.
4. Influence of Changing Sample Size (Chapter IV of text) Problems 32, 33, 34 and 35

Non-Sampling Errors in Sampling

PROBLEM 24. SAMPLING PLANS AND BIASES

1. A survey was made by a research agency for a manufacturer of garden tools to discover whether it might pay to put on an intensive advertising campaign directed to city dwellers. Personal interviews were used in households selected at random.

Instructions to the interviewers read, in part, "If no one is at home when you call, substitute for that call the nearest household at which you do find someone at home." The key question asked in all completed interviews was, "Do you have a vegetable garden?"

Question

What opportunities for bias do you see in this instruction to the interviewers?

2. When personal interviews or questionnaires are used to determine magazine readership, there is a bias in favor of the prestige magazines such as *Fortune*, *Atlantic Monthly*, *Harper's*, *Holiday*, *Saturday Review of Literature*, etc.

One agency, to avoid this bias, posed as a waste paper buyer and went through the survey area paying 2¢ a pound for old magazines. Each of the households from which purchases were made was classified by its estimated income level and a record was made of magazines found in each income class.

Question

Is it possible that the agency avoided one source of bias at the expense of introducing another bias?

3. When making a market area survey, one question asked in personal interviews was, "What proportion of your clothing purchases is made out of town?" The sample was designed for 200 interviews selected at random.

Ten per cent of those interviewed replied "I don't know," "It is none of your business," or made similar evasive answers.

The interviewers argued that only 20 persons made such replies and this was not enough to trouble about. The supervisor insisted, however, that 25 additional names be selected at random and the quota of 200 completed interviews be made by finding among the 25 additional persons 20 who would cooperate.

Questions

a. What bias do you think might have resulted from using the original 180 interviews for the analysis?

b. Do you think the supervisor's action in taking additional interviews removed the bias?

c. What should have been done?

4. Taxes on real property provide a large part of the money used to finance local government, schools, and municipal facilities. Real estate is supposed to be valued at its full market value, but in

PROBLEM 24. (Continued)

periods of rapidly changing prices, such as 1940–1950, tax values and market values are likely to get wide apart and the relation is different from property to property and community to community with resulting inequities in the tax burden.

In some states it is provided that properties which are sold shall have their sales prices compared with their tax values (tax values ÷ sales values); these ratios are then averaged and tax values in that area are then raised or lowered to equalize tax and market values.

Questions

a. Is a sample taken in making this adjustment? Is it a random sample? What kind of sample is it? Discuss.

b. Would you suspect bias in the tax value-market price ratios because of the method of selection? Explain.

PROBLEM 25. MISTAKES, COMPENSATING ERRORS, AND BIAS

In measuring the outer diameter of Saturn's ring, the astronomer Bessel secured the following results at Königsberg Observatory. Each item represents an individual reading of the distance in seconds. The arithmetic average of the items, 39″.308, was taken as the best approximation of the distance.

TABLE 7

BESSEL'S READINGS OF THE OUTER DIAMETER OF SATURN'S RING

38″.91	39″.32	38″.93	39″.31
39 .17	39 .04	39 .57	39 .46
39 .30	39 .03	39 .35	39 .25
39 .14	39 .47	39 .29	39 .32
39 .40	39 .33	39 .28	39 .62
39 .41	39 .40	39 .36	39 .20
39 .42	39 .30	39 .41	39 .43
39 .43	39 .36	39 .02	39 .01
38 .86	39 .51	39 .21	39 .17
39 .60	39 .54	39 .45	39 .72

Source: *Spherical and Practical Astronomy*, Chauvenet, Lippincott & Co., 1868, Vol. II, p. 495.

Questions

1. As will be noticed, 40 different measurements of the distance were made and identical results obtained but twice. Why is there this variation among the repeated measurements of the same thing?

2. In view of your discussion under 1, is the failure to get the same answer repeatedly due to a mistake, random errors, or bias?

3. What is the justification of taking the arithmetic average and permitting it to represent the true distance?

4. Does Bessel's method mean that when our statistical results fail to "check out" we can take an average of our incorrect results and obtain the right answer? If so, under what circumstances; if not, why not?

PROBLEM 26. MISTAKES, COMPENSATING ERRORS, AND BIAS

Table 8 comes from the census of population taken by the Bureau of the Census in 1940.

TABLE 8

AGE DISTRIBUTION OF THE POPULATION, UNITED STATES, 1940

Age Group	*Number*		*Per Cent*	
	Male	*Female*	*Male*	*Female*
All Ages	*66,061,592*	*65,607,683*	*100.0*	*100.0*
Under 5 years	5,354,808	5,186,716	8.1	7.9
5 to 9 years	5,418,823	5,265,799	8.2	8.0
10 to 14 years	5,952,329	5,793,606	9.0	8.8
15 to 19 years	6,180,153	6,153,370	9.4	9.4
20 to 24 years	5,692,392	5,895,443	8.6	8.9
25 to 29 years	5,450,662	5,645,976	8.3	8.6
30 to 34 years	5,070,312	5,172,076	7.7	7.9
35 to 39 years	4,745,659	4,799,718	7.2	7.3
40 to 44 years	4,419,135	4,368,708	6.7	6.6
45 to 49 years	4,209,269	4,045,956	6.4	6.2
50 to 54 years	3,752,750	3,504,096	5.7	5.3
55 to 59 years	3,011,364	2,832,501	4.6	4.3
60 to 64 years	2,397,816	2,330,524	3.6	3.6
65 to 69 years	1,896,088	1,910,569	2.9	2.9
70 to 74 years	1,270,967	1,298,565	1.9	2.0
75 years and over	1,239,065	1,404,060	1.9	2.1

Source: *Statistical Abstract of the United States*, 1949, p. 11, Department of Commerce, Bureau of the Census.

Notice that the total number of females is somewhat less than the number of males and that this relationship holds until the age bracket 20 to 24 years is reached, then until age 40 the females appear to be more numerous than the males. This is true both in terms of total numbers and the proportion of each sex within the age group.

Between the ages of 40 and 60, however, the proportion of females is less than found in the male population. In old age the females again exceed the males both in numbers and the proportion surviving.

Questions

1. Do the characteristics of these data, pointed out in the two preceding paragraphs, seem reasonable to you on *a priori* grounds?

2. Can you pick up any clues which lead you to think that mistakes, compensating errors, or bias have entered in the collection, tabulation, or publication of these data? Be explicit.

3. Could any errors or mistakes you have detected be eliminated by a process of averaging?

PROBLEM 27. FORECAST CHANGE IN AGE DISTRIBUTION

TABLE 9

FORECAST AGE DISTRIBUTION OF THE POPULATION
UNITED STATES IN THE YEAR 2000

Age Group	*Number*	*Per Cent*
Total	*163,312,000*	
Under 5 years	9,847,000	6.0
5 to 9 years	10,645,000	6.5
10 to 14 years	10,702,000	6.6
15 to 19 years	10,812,000	6.6
20 to 24 years	11,129,000	6.8
25 to 29 years	11,546,000	7.1
30 to 34 years	11,663,000	7.1
35 to 39 years	11,271,000	6.9
40 to 44 years	10,853,000	6.6
45 to 49 years	11,011,000	6.7
50 to 54 years	11,620,000	7.1
55 to 59 years	11,874,000	7.3
60 to 64 years	8,831,000	5.4
65 to 69 years	7,158,000	4.4
70 to 74 years	6,189,000	3.8
75 years and over	8,161,000	5.0

Source: *Statistical Abstract of the United States*, 1948, p. 29. Department of Commerce, Bureau of the Census.

Instructions

1. Refer to Problem 26 and write a brief statement relating to the important changes anticipated in the number and age distribution of the population of the U. S.

Questions

1. List briefly some of the economic consequences of the shifts analyzed above.
2. Appraise the reliability of such forecasts as this.

Selecting a Sample

PROBLEM 28. THE CHOICE OF A SIMPLE RANDOM SAMPLE

In Table 10 will be found the cost of meals in the à la carte division of the University Commons during the luncheon hour, Thursday, December 11, 1941. The checks are listed in the order recorded on the tape in the cash register. In Table 11 is reported the same type of data taken in the same way for luncheon, March 9, 1950.

Instructions

1. (*a*) Select two random samples from the luncheon data for December 11, 1941. The first sample should contain not less than 35 items, and the second sample should be at least twice as large.
   (*b*) Compute the arithmetic mean of each of your samples.
2. (*a*) Carry out instructions 1*a* and 1*b* above, but use data for March 9, 1950.

(Note: Save copies of the 2 large samples as these are to be used in Problem 46.)

PROBLEM 28. (Continued)

Questions

1. Table 10 includes 384 items while Table 11 contains 650 items. Should the sample of the Table 11 data be larger than the sample of Table 10 data in the ratio 650/384 to obtain equally reliable results? Explain.

2. What methods did you use to obtain random choices in selecting items for your samples? Explain.

3. Should the sample of the 1950 data be larger than the sample of the 1941 data because the average expenditure in 1950 is higher?

4. If you took another sample of the same size as your larger sample from the same universe would you expect to obtain an identical average? Explain.

5. Do you think your sampling results are reasonably satisfactory? Explain.

6. Select one of your samples and explain how many additional items would be needed to halve the random sampling error.

7. It is clear that luncheon costs were much higher in 1950 than in 1941. Would you, however, assign increased prices as the sole cause of the differences in the averages you obtained? If not the sole cause, give other reasons.

8. Would the fact that one set of data was taken in December and the other in March make any difference? Explain.

PROBLEM 28. (Continued)

TABLE 10

COST OF MEALS SERVED AT THE UNIVERSITY COMMONS, À LA CARTE, LUNCHEON HOUR, DECEMBER 11, 1941
(384 items)

20	36	48	55	23	29	33	46	35	30
25	44	42	31	47	41	25	55	26	34
47	34	18	23	47	23	23	42	39	49
37	24	30	25	36	26	18	38	18	52
23	28	26	34	25	30	20	40	24	42
29	25	28	36	48	44	25	48	52	32
31	29	46	35	36	24	31	30	29	33
34	24	39	42	32	32	48	48	45	30
23	36	12	39	40	28	43	52	30	45
38	32	12	15	33	32	32	50	42	39
44	45	25	29	40	12	43	49	54	33
36	30	56	35	41	45	62	46	48	30
46	55	26	51	39	43	52	44	25	19
54	45	55	52	58	52	41	40	30	36
27	50	37	29	31	32	35	44	24	30
37	43	59	46	53	31	29	33	35	47
30	49	43	36	37	29	53	34	19	43
21	32	33	23	38	49	44	39	25	54
32	27	31	22	27	48	21	59	40	39
28	32	35	31	40	47	44	24	25	15
25	39	21	30	35	23	49	32	48	23
44	34	35	38	40	41	54	33	36	42
18	43	21	43	39	26	24	32	26	35
42	23	40	38	41	28	33	40	56	27
47	17	45	30	54	29	50	38	36	
37	42	50	37	29	33	15	42	43	
37	23	36	28	26	50	24	27	22	
15	25	35	23	29	48	32	38	26	
31	33	36	21	28	46	30	19	29	
58	29	23	28	45	31	50	33	29	
22	28	26	51	34	36	33	26	37	
34	29	38	28	30	36	52	42	50	
29	45	38	39	39	46	38	11	36	
18	35	47	18	30	40	50	36	28	
24	37	33	18	30	41	27	64	34	
39	46	27	35	33	48	25	36	34	
32	43	36	29	44	19	32	50	34	
25	42	32	45	43	15	19	43	23	
38	54	20	29	19	18	50	12	29	
44	52	29	19	35	10	45	12	31	

PROBLEM 28. (Continued)

TABLE 11

COST OF MEALS SERVED AT THE UNIVERSITY COMMONS, À LA CARTE, LUNCHEON HOUR, MARCH 9, 1950
(650 Items)

67	64	64	17	59	71	47	92	71	51	53	24	40
43	32	64	60	28	50	69	66	58	86	60	64	70
84	59	19	87	57	40	63	45	66	31	48	55	40
60	64	19	45	76	59	74	50	63	41	60	70	78
66	83	57	57	62	35	67	59	60	55	58	44	52
87	71	40	61	61	51	79	62	54	62	67	47	64
45	62	78	52	65	39	60	62	72	60	69	45	48
48	89	47	43	68	1.08	80	80	43	64	63	83	75
81	66	51	58	55	56	69	21	45	32	66	25	73
61	57	68	68	55	72	64	71	49	73	67	25	56
64	57	53	47	69	53	56	70	69	55	78	66	45
59	79	52	55	62	71	81	68	60	41	89	85	74
63	1.03	48	62	72	75	50	69	45	29	46	72	65
62	65	54	61	59	73	68	52	57	37	62	81	66
81	80	59	27	84	74	48	41	72	41	43	40	63
55	42	57	38	71	72	76	49	47	67	71	66	44
70	64	67	51	50	67	17	45	86	62	77	33	53
86	64	70	67	67	71	58	85	66	90	87	57	33
74	60	67	69	55	52	74	45	67	48	63	45	58
60	77	70	62	59	77	74	55	57	54	95	67	63
76	91	54	69	57	79	55	77	65	15	61	64	62
34	40	75	46	64	78	64	68	51	71	20	75	40
80	74	51	55	62	62	62	69	62	49	82	53	60
79	67	65	57	53	82	65	80	50	87	74	58	63
75	67	59	79	60	74	79	61	40	54	67	75	35
67	62	72	56	53	70	46	80	63	58	75	87	47
71	64	64	77	84	53	44	60	64	67	62	55	55
60	38	62	55	70	59	46	65	60	61	38	62	19
74	45	48	67	46	48	67	67	74	93	68	67	74
90	52	61	55	83	69	81	57	62	57	86	57	67
74	49	57	68	69	57	62	74	45	59	73	75	48
53	61	54	57	60	78	55	68	67	52	50	53	36
60	57	46	57	50	50	70	53	57	46	70	89	50
70	90	65	59	61	64	55	51	60	54	86	51	68
77	44	58	51	73	73	63	57	66	82	65	69	69
81	86	61	62	49	58	69	62	59	28	56	43	68
72	90	67	83	55	69	64	57	58	36	1.21	59	71
60	60	50	45	53	52	61	61	57	48	45	70	54
73	71	50	53	50	41	52	59	62	55	72	66	72
87	86	69	62	57	82	65	40	62	32	68	34	59
60	69	63	46	72	68	25	25	73	51	65	1.07	86
72	46	64	46	50	66	57	43	59	32	20	55	75
73	72	57	58	65	80	57	66	67	72	45	70	82
88	49	55	48	53	64	40	26	40	64	64	69	89
52	75	51	58	43	67	45	75	73	57	70	74	71
72	79	79	85	50	74	66	76	67	41	53	65	55
61	70	90	65	64	62	45	57	76	59	65	79	69
53	1.04	60	64	57	70	60	64	45	72	57	65	58
75	85	88	81	61	76	72	45	83	84	69	53	47
64	62	60	54	42	73	81	67	55	72	49	75	56

PROBLEM 29. THE CHOICE OF A SIMPLE RANDOM SAMPLE

The monthly consumption of electricity by 508 domestic users is listed in Table 12. The kilowatt consumption recorded in the table has been taken directly from the meter readers' books and represent three "routes" assigned to meter readers in different parts of the city selected so as to avoid the poorest parts and the industrial-commercial areas. In other words, these records of consumption come from one stratum of electricity users—the middle-income domestic users.

It is important to notice, too, that the kilowatt-hours used have been taken directly from the meter readers' books without rearrangement. To the extent larger or smaller users tend to be grouped within blocks or along certain streets on the routes, those groupings will be present in the listings in the table.

Instructions

1. Consider the method of collecting and listing the data in Table 12 and decide how to select two simple random samples from the list. Write a brief explanation of the method or methods you use.
2. Take the two simple random samples. Make the sample size suitable but make the two samples differ in size. Save your work sheets for use in a later problem (Problem 46A) where the individual values in the larger of your two samples will be grouped in frequency distribution form.
3. Compute and compare the arithmetic means of your two samples.

Questions

1. What is the population you are sampling?
2. Suppose you had taken the first 50 items in the list for one of your samples. Would this have been a random sample of the population? Explain.
3. How did you assure a random sample in your first drawing? In your second drawing? Explain.
4. Are your two samples independent? Explain.
5. How do you explain any difference which appears between the arithmetic means of your first and second samples? If the two samples had been of the same size would you then expect their means to be identical also?
6. Select one of your random samples and explain how many additional items would be needed in your sample to reduce the random errors by one half.
7. Do you see any opportunity for biases to have crept into your sample? If so, explain.

PROBLEM 29. (Continued)

TABLE 12

MONTHLY KILOWATT-HOUR CONSUMPTION OF ELECTRICITY, SMALL DOMESTIC USERS
ILLINOIS POWER COMPANY, 1950

110	94	240	100	130	246	90	44	152	168	170	120	48
96	110	162	120	34	122	88	10	90	162	96	108	38
14	180	78	202	78	282	134	70	94	170	114	64	220
280	246	76	122	102	150	320	74	84	188	164	110	138
100	86	46	118	94	64	324	316	88	84	262	302	16
84	68	96	42	46	136	84	192	208	100	48	32	150
316	56	68	138	186	70	102	146	140	52	156	70	58
122	102	102	180	64	316	74	198	78	130	100	224	76
130	258	42	78	36	140	56	262	80	98	24	146	142
60	154	86	112	6	138	158	92	198	288	48	140	72
78	110	156	96	86	246	104	216	172	88	52	186	90
34	94	52	62	122	88	106	82	172	58	114	276	30
68	174	38	20	208	466	62	144	86	306	110	174	80
170	210	92	6	54	54	208	400	134	58	374	140	90
104	114	82	148	106	102	110	100	88	82	114	192	152
90	90	104	180	34	220	64	174	68	48	146	146	18
56	296	64	224	48	74	76	174	60	102	82	130	190
232	82	62	96	126	76	142	84	184	60	62	102	110
38	4	80	172	70	96	138	52	64	124	96	140	354
90	92	70	84	128	74	98	140	390	284	296	136	82
78	144	94	102	116	128	96	138	78	60	32	78	212
116	130	50	62	180	74	80	60	128	74	52	68	150
144	96	80	142	92	52	92	298	40	70	120	98	138
88	76	152	62	276	276	162	112	56	118	82	16	84
134	94	172	98	140	36	246	60	84	100	142	75	110
56	82	138	178	62	44	122	102	130	162	94	160	162
144	262	112	31	116	124	82	176	200	140	112	82	130
52	146	230	02	180	460	96	82	110	228	62	94	198
358	36	86	210	164	146	180	78	48	242	146	40	246
72	118	136	86	70	98	32	28	116	80	98	20	176
226	10	226	164	84	138	84	110	144	130	116	590	104
146	100	364	164	46	116	106	34	78	150	18	28	28
280	176	172	136	66	110	42	64	194	80	208	52	68
42	66	142	154	58	70	116	78	152	244	14	46	230
150	92	112	106	66	76	194	78	200	128	194	262	154
138	116	66	74	256	34	108	154	150	82	216	44	112
70	55	172	56	44	70	136	64	84	50	142	64	
94	96	108	56	258	72	110	76	388	50	62	216	
342	124	42	364	182	168	36	90	282	136	78	300	
144	86	106	96	210	120	82	46	172	46	78	44	
350	88	28	256	94	168	62	82	78	196	72	186	
194	54	94	110	170	66	144	88	102	114	128	72	
110	96	36	182	144	88	124	66	102	98	140	74	
126	30	14	80	110	136	36	26	108	48	28	78	
136	124	56	62	78	104	38	52	118	100	118	26	
62	154	90	76	98	44	122	68	132	74	50	36	

Source: The Illinois Power Company, 1950.

PROBLEM 30. DESIGN OF A SAMPLE

In Problem 22 you prepared a schedule to be used in analyzing the market for fountain pens on the University campus.

Instructions

1. Design a sample to be taken of the student body to which you belong for a cost of $200.00.

Questions

1. What kind of a sample would you call the one you have designed?
2. Is your sample a random sample? Explain how the random selections are assured.
3. If you have employed stratifications, justify each of the stratifying principles used.
4. Is it your opinion that each student would have an equal chance with every other for inclusion in your sample? If not, justify any departure from this standard.

PROBLEM 31. DESIGN OF A SAMPLE

In the library will be found the 1940 Census of Population. Among the volumes reporting population are those on "Population and Housing" and supplementary volumes on "Housing." Each city in the United States with a population of 250,000 or over is divided into tracts with about equal population and "block" statistics are provided for these and smaller cities as well. These data, collected by the government, are an aid to research workers in designing samples in American cities.

Instructions

1. Choose a project the completion of which would require a survey of households in a selected metropolitan center. State your problem with sufficient care to guide you in designing a sample.
2. Select a city for your analysis and locate the city, its map, census tracts, if any, and block statistics in the Census volumes.
3. Assume that you have $2,000 for the field survey and design the sample.
4. Explain fully the sample design and show how the tracts (if any), block and households would be selected. (It is not required that the households be actually drawn; merely explain *how* you would select them.)
5. Show how you have assured a random selection at all levels of selection and if stratifications are employed, justify your choice of stratifying principles.

The Influence of Changing Sample Size

PROBLEM 32. THE PRINCIPLE OF DECREASING VARIATION

The data in Table 13 are arithmetic averages computed from simple random samples of varying sizes drawn from data in Table 11 — amounts of checks paid at the University Commons, à la carte division, luncheon hour, March 9, 1950. Each sample is independent.

PROBLEM 32. (Continued)

TABLE 13

ARITHMETIC AVERAGES OF CHECKS PAID, SAMPLES OF VARIOUS SIZES AND DIFFERENCES IN SUCCEEDING AVERAGES OF SAMPLES. À LA CARTE DIVISION, LUNCHEON HOUR, UNIVERSITY COMMONS, THURSDAY, MARCH 9, 1950

Number of Items in Sample	*Arithmetic Average Checks Paid and Difference between Succeeding Averages*					
	A Group		*B Group*		*C Group*	
	Ave. in Cents	*Diff. in Cents*	*Ave. in Cents*	*Diff. in Cents*	*Ave. in Cents*	*Diff. in Cents*
25	55.1		64.3		60.6	
50	58.5	+3.4	59.4	−4.9	62.2	+1.6
100	60.6	+2.1	61.8	+2.4	62.7	+0.5
200	61.3	+0.7	62.1	+0.3	60.3	−2.4

Arithmetic average of 650 items—61.4 cents

Source: Random samples from Table 11.

Questions

1. Examine groups A, B, and C and note what tends to happen to the differences between the arithmetic averages as the sample is enlarged. Why should this be so?

2. Are the differences in Group C an exception to the tendency that the variation between averages decreases as the sample is enlarged? Why?

3. Since the samples are drawn from a single universe, how do you explain the variation among the averages of samples of given size?

4. In so far as you can judge from the evidence here presented, do you think a sample of 25 large enough? of 50? of 100? of 200? Explain your reasons.

PROBLEM 33. THE PRINCIPLE OF DECREASING VARIATION

The data of Table 13 have been rearranged and one additional set of results introduced again to illustrate the principle of decreasing variation in sampling results.

TABLE 14

VARIATION AMONG ARITHMETIC AVERAGES COMPUTED FROM SAMPLES OF GIVEN SIZE

	25-Item Sample	*50-Item Sample*	*100-Item Sample*	*200-Item Sample*
First sample	55.1	58.5	60.6	61.3
Second sample	64.3	59.4	61.8	62.1
Third sample	62.7	59.4	61.1	61.8
Fourth sample	60.6	62.2	62.7	60.3
Range	9.2	3.7	2.1	1.8

Arithmetic mean of 650 items—61.4 cents

Source: Simple random samples from Table 11.

Questions

1. After study of these data, do you find it necessary to revise the decisions made in answer to Question 4, Problem 32?

2. Do the data reveal increasing stability in the arithmetic averages of successively larger random samples? Explain briefly.

PROBLEM 34. SAMPLING BY "TAKING THEM AS THEY COME." A CUMULATIVE SAMPLE

The averages in Table 15 are computed from samples drawn from the data in Table 11, amounts of checks paid at the university commons, luncheon hour, March 9, 1950. The samples represented were "taken as they come" — the first 25, then another 25 which were combined with the first 25 to make a sample of 50, and so on until a sample of 200 was taken. In other words, the samples are cumulative, and are not independent.

TABLE 15

AVERAGE CHECKS PAID FOUR SAMPLES AND DIFFERENCES BETWEEN SUCCEEDING AVERAGES OF SAMPLE

Items in Sample	*Averages in Cents*	*Differences in Cents*
25	66.4	
50	68.0	+1.6
100	67.4	−0.6
200	62.5	−4.9

Average of 650 items—61.4 cents

Source: Table 11.

Instructions

1. Compare the arithmetic means of the samples of varying size.

Questions

1. What do you understand are the main differences in the way the samples reported here were taken and in the way the samples shown in the previous two problems were taken?
2. Do these results exhibit the tendency toward decreasing variation?
3. Do you recommend "taking them as they come" as a satisfactory method of sampling? Why?
4. Would you recommend the cumulative type of sample used above as a satisfactory method of sampling? Why?

PROBLEM 35. REVIEW QUESTIONS ON SAMPLING

1. "A sample is never so satisfactory or practical as a complete enumeration." Do you agree? Explain.
2. What sampling tendencies have been illustrated in Problems 32–34? Write out a brief statement explaining in general terms the meaning of each of these tendencies.
3. It would be necessary to take a larger sample to determine the arithmetic average of meat purchases by urban families than to determine their bread purchases. Why?
4. "Stability of sampling results depends on extent of variability in the universe." Explain.
5. Distinguish carefully among random errors, bias, and mistakes.
6. Explain how random errors tend to distribute themselves.
7. "Bias results from dishonesty in sampling." Do you agree? Explain.
8. "Simple random sampling" means "taking them as they come." Do you agree? Explain.

SECTION IV

The Presentation of Statistical Data

1. The Construction of Tables (Chapter V of text) Problems 36, 37, 38
2. The Construction of Charts — Arithmetic Scales (Chapters VI and VII of text) Problems 39, 40, 41, 42
3. The Construction of Charts — Semi-Logarithmic Scale (Chapter VII of text) Problems 43, 44
4. Summary and Review, Problem 45

THE PRESENTATION OF STATISTICAL DATA

Tabular Forms

PROBLEM 36. SIMPLE TABULAR FORM

The *Survey of Current Business*, published by the U. S. Department of Commerce, reports in its issue for February 1950 that:

The sharpest decline in manufacturing activity during 1949 was in the heavy goods industries, which were affected by both inventory adjustment and the slackening of demand for producers' durable goods.

The effect of the decline on business outlays for capital equipment was evident in the production trends of all producers' durables, including especially machine tools, most types of industrial machinery, transportation equipment, mining and oil well equipment, and integral and fractional horsepower motors.

Exceptions to the decline were found in heavy electrical equipment production.

The value of manufacturers' sales for all manufacturing was $205.1 billion in 1947. These sales were $228.0 and $213.4 billions in 1948 and 1949 respectively.

Durable-goods industries reported total sales of $82.6 billions in 1947, $94.7 billions in 1948, and $89.1 billions in 1949. Durable-goods production amounted to 40.3% of the value of all manufactures in 1947, rose to 41.5% in 1948, and to 41.8% in 1949.

In the nondurable goods segment production has been considerably more stable. Nondurable-goods industries reported total sales of $122.5 billions in 1947, and of $133.3 and $124.3 billions of sales in 1948 and 1949 respectively. Production of nondurable goods contributed 59.7% in 1947, 58.5% in 1948 and 58.2% in 1949 of all manufacturers' sales for the respective years.

No adjustment in dollar values has been made in any category for price level changes.

Instructions

1. Construct a table with proper headings, captions, stubs, totals, footnotes and sources, which will facilitate a comparison of the above data. Number your table, Table 1. (Be sure to utilize all totals, subtotals, and percentages in your table.)
2. Examine the order of emphasis which you have given the data and justify it.

PROBLEM 37. CONSTRUCTION OF A TABLE

Veterans desiring to rent homes are able to pay: less than $30 per month for rent and utilities, 8 per cent; $30 to $39, 26 per cent; $40 to $49, 36 per cent; $50 to $59, 19 per cent; $60 and over, 11 per cent.

Source: *Veterans Housing Plans and Living Arrangements in 1946*, for 108 survey areas. Statistics Bulletin No. 2, January, 1948. Housing and Home Finance Agency, Washington.

Instructions

1. Construct a table with proper headings, captions, stubs, totals, footnotes and sources. (Number your table, Table 2.)
2. Examine the order of emphasis you have given and justify it.

PROBLEM 38. CONSTRUCTION OF A CROSS-CLASSIFIED TABLE

Consumer Credit, Inc., operates lending agencies in Sacramento, Riverside, and San Diego, California. Three types of loans are made—automobile, electrical appliance (these two are on installment sales contracts) and personal loans.

The manager in the home office asks the accounting department to show all past due accounts classified by "under 30 days;" "30 to 60 days," and "over 60 days." The balance due on past due accounts is also required, that is, "under $300," "$300 to $900," and "over $900." These breakdowns of number and value of past due accounts are to be shown in one table cross classified by offices and type of loan, i.e., automobile, appliance and personal.

Instructions

1. Draw up the table form which will cross classify by the four characteristics. Show all totals and subtotals. Number your table, Table 3.

2. Test the adequacy of your table by classifying the following with but one entry in the table —an automobile loan, Riverside office, overdue 15 days, amount due $750.00.

The Construction of Charts

PROBLEM 39. COMPARISON OF SIMPLE MAGNITUDES

TABLE 16

BUDGET RECEIPTS AND EXPENDITURES, THE FEDERAL BUDGET, UNITED STATES, SELECTED YEARS (Billions of Dollars)

Fiscal Years	*Receipts*	*Expenditures*	*Surplus or Deficit*
1939	5.1	9.0	−3.9
1947	43.2	42.5	.7
1948	42.2	33.8	8.4
1949	38.2	40.0	−1.8
1950*	37.8	43.3	−5.5
1951*	37.3	42.2	−5.1

* Estimates.

Source: *The Federal Budget in Brief*, Fiscal Year 1951. Executive Office of the President, Bureau of the Budget (fiscal year extends July 1 to June 30).

Instructions

1. Make a chart which will show receipts, expenditures, and surplus or deficit of the Federal government as recorded in Table 16. Number your chart, Chart 1.

Questions

1. What is the percentage increase in Federal expenditures between 1939 and 1949?

2. In 1939, the National Income was $72.5 billion; in 1949 it was approximately $224 billion. On the basis of these data would you say the Federal government has taken a larger or smaller portion of the National Income in 1949 as compared with 1939?

3. It is sometimes argued that the Federal government should follow the policy of "compensatory" spending. That is, when expenditures in the "private" sectors of the economy decrease and business activity declines, then the Federal government should increase expenditures in the "public" sector—create a deficit in the hope of stabilizing or expanding business acivity. The opposite actions

PROBLEM 39. (Continued)

would be taken when expenditures by the "private" sectors of the economy are expanding at high levels.

Whether "compensatory" spending can do what is expected of it may be a question. Do these data indicate, however, that an attempt has been made to follow this policy? Are you satisfied that you have enough evidence to support your position?

PROBLEM 40. COMPONENT PARTS

TABLE 17

GROSS NATIONAL PRODUCT, UNITED STATES, 1939–1949

	In Billions of Dollars		*In Per Cent*	
	*1949**	*1939*	*1949*	*1939*
Gross national product	263	91	100	100
Personal consumption	178	67	68.0	74.0
Private domestic investment	41	10	15.6	11.0
Net foreign investment	1	1	.4	1.0
Government purchases				
Federal government	25	5	9.5	5.0
State and local governments	18	8	6.8	9.0

* First quarter estimates at annual rates.

Source: *Survey of Current Business*, 1949 in January 1950 issue, p. S-1; 1939 values from *Statistical Supplement*, p. 7.

The gross national product represents the total value, in current dollars, created by the productive activity of the economy.

Individuals purchase by far the largest part of the product for their personal consumption but a large value of goods and services goes into new homes, industrial plant, equipment and inventories classified under "private domestic investment." Foreign countries buy a part of the product and also sell goods and services to the United States, gifts and payments flow both in and out, and the difference represents net foreign investment. Government competes with each of the other groups in buying goods and services.

Instructions

1. Make one or more charts showing graphically the decade change in GNP and its distribution among the four categories of expenditure. Number your chart or charts, Chart 2; 2A.

Questions

1. If it were necessary to choose between charting the percentages and the absolute amounts, which would you choose and why?
2. Could any of the data be plotted satisfactorily as a pie diagram? Explain.
3. Write a brief paragraph explaining the changes in the distribution of the GNP among the classes of buyers.
4. Would the observed changes in the distribution of the GNP explained in 3 above, in any way explain the inflation between 1939 and 1949?
5. The level of prices in the United States increased about 70 per cent between 1939 and 1949. How does this statistic help you decide whether the apparent increase in GNP was real or merely a repricing of goods and services as a result of inflation?

PROBLEM 41. COMPONENT PARTS

During the 34 years, 1914–1948, the people of the United States shipped abroad an estimated $101,000,000,000 of goods and services in excess of the quantities foreign enterprise shipped to the United States. This is sometimes referred to as a "favorable balance of trade." About half of the $101 billion represents goods which went to allies during the first and second World Wars.

The question arises, how did the foreign buyers get the dollars to pay for this $101 billion in excess of their commercial transactions with the United States? An answer is found in Table 18.

TABLE 18

SOURCE OF DOLLARS TO COVER DEFICITS WITH UNITED STATES, 1914–1948

	Billions of Dollars	*Per Cent*
Private donations to individuals living abroad	10.5	10.0
Private investments made by U. S. citizens abroad	10.5	10.0
Gold shipped to U. S. to settle balances	15.5	14.8
U. S. Government loans and credits to abroad		
Grants	49.0	46.9
Loans	19.0	18.3
Total	104.5	100.0

Source: Estimate imputed to the State Department and reported in Press during November 1949. See, for example, *The Philadelphia Inquirer*, November 28, 1949.

Instructions

1. Note that the total of table values does not equal the total reported in introductory paragraphs. The discrepancy is said to be due to "duplications on which the bookkeeping has not yet been done." Make a decision as to what should be done about this inconsistency.

2. Construct a chart which, in your opinion, adequately pictures the data. Number your chart, Chart 3.

PROBLEM 42. SEARS ROEBUCK AND COMPANY SALES

TABLE 19

AVERAGE MONTHLY SALES, SEARS, ROEBUCK AND COMPANY, 1915–1948

(Mail Order and Store Sales in Thousands of Dollars)

Year	*Sales*	*Year*	*Sales*	*Year*	*Sales*
1915	$ 9,389	1930	$32,532	1945	$88,162
1916	12,237	1931	28,934	1946	138,872
1917	14,856	1932	23,339	1947	174,146
1918	16,544	1933	23,714	1948	201,485
1919	21,494	1934	28,075		
1920	21,216	1935	34,460		
1921	14,835	1936	43,447		
1922	15,180	1937	47,818		
1923	17,962	1938	44,407		
1924	18,515	1939	54,233		
1925	21,529	1940	61,657		
1926	22,725	1941	79,671		
1927	24,411	1942	76,489		
1928	28,914	1943	72,750		
1929	36,954	1944	83,386		

Source: 1915–1940 data from *Survey of Current Business*. U. S. Department of Commerce, 1940 Annual Supplement, p. 31: February 1941, p. 71: December 1941, p. S-7. Statistical Supplement, 1949, p. 51.

Instructions

1. Construct a chart showing the variations in Sears, Roebuck and Company sales for the period 1915–1948. Number your chart, Chart 4.

Questions

1. Are you able to see, by referring to your chart, the business cycle influence on these data? If so, what dates would you specify as troughs and which peaks of activity?

2. Is the seasonal variation apparent in these data? If so, describe the seasonal movement. If not, why is it not apparent?

3. The rapid growth of this company should be clearly apparent in your chart. There is a chart in Chapter VII, page 181, of the text showing Sears, Roebuck and Company sales, 1924–1940, when the growth factor has been computed and eliminated from the sales. This chart may be examined and compared with Chart 4. The method of making such adjustment is explained in Chapter XV of the text.

4. Notice that you have plotted average monthly sales. How would a curve of total annual sales differ from the one you have constructed?

5. On the basis of the text discussion and your personal knowledge of this company's history, how do you explain the cyclical variation? The rapid growth?

PROBLEM 43. COMPARING RATES OF CHANGE—THE USE OF THE RATIO SCALE

The rapid growth in air transport in the United States is best seen in comparison with the growth shown by a competitive and widely used method of transportation. Millions of passenger miles traveled in Pullman cars on domestic railroads and millions of passenger miles traveled in scheduled domestic airlines are shown in Table 20.

TABLE 20

PASSENGER MILES
PULLMAN CAR AND DOMESTIC AIRLINES
UNITED STATES, 1930–1947

Year	*Passenger Miles in Millions*	
	Pullman	*Airlines*
1930	12,515	85
1931	9,892	106
1932	6,758	127
1933	6,142	175
1934	6,891	190
1935	7,146	316
1936	8,355	439
1937	9,170	481
1938	8,270	561
1939	8,485	755
1940	8,214	1,158
1941	10,070	1,506
1942	19,072	1,501
1943	25,891	1,671
1944	28,267	2,212
1945	27,276	3,408
1946	20,672	6,068
1947	13,000	6,308

Source: *Statistical Handbook of Civil Aviation*, Civil Aeronautics Administration, 1948, p. 79.

Instructions

1. Examine the data and decide how many cycles will be needed on the ratio scale to accommodate them. Use the ratio paper found in this Manual and plot the series. Number the chart, Chart 5.

Questions

1. Do either of the series seem to follow a reasonably constant rate of growth per year? Explain. (Visual analysis of the plotted data should enable you to answer this question.)
2. Which of the series shows the more rapid rate of increase? How do you know?
3. During which periods of time did the airlines improve their competitive position most rapidly? Explain.
4. By visual analysis alone, determine how the Pullman series is changing; i.e., increasing at a decreasing rate, increasing at an increasing rate, increasing at a constant rate, etc. Write a brief paragraph in explanation.
5. What information can you obtain from this chart by visual analysis which you could not obtain from an arithmetic scaling of the same data?
6. On the basis of present trends, would you venture a prediction as to the approximate date at which the lines of the two curves might be expected to cross? Give reasons in support of your forecast or in support of your refusal to make such a forecast.

PROBLEM 44. COMPARING VALUES OF WIDELY DIFFERING MAGNITUDES — THE USE OF THE RATIO SCALE

In Table 21 will be found a record of gasoline consumption in the United States and in the State of Illinois, 1925–1948. One use of the ratio (semi-logarithmic) scale is to facilitate comparisons of such series as these where the magnitudes of the numbers are quite different.

TABLE 21

GASOLINE CONSUMPTION UNITED STATES AND ILLINOIS
1925–1948

(Millions of Gallons)

	U.S.	*Illinois*
1925	9,144	584
1926	10,552	660
1927	11,937	722
1928	13,090	791
1929	14,678	845
1930	15,762	973
1931	16,719	1,048
1932	15,497	951
1933	15,436	971
1934	16,595	1,026
1935	17,742	1,069
1936	19,699	1,192
1937	21,232	1,293
1938	21,419	1,332
1939	22,678	1,420
1940	24,128	1,510
1941	26,975	1,637
1942	23,630	1,416
1943	21,813	1,165
1944	24,334	1,166
1945	24,435	1,273
1946	30,077	1,644
1947	32,733	1,810
1948	35,532	1,971

Source: 1925–1928 *Highway Statistics*, Summary to 1945 Public Roads Administration, p. 6; 1929–1948 American Petroleum Institute, various releases.

Instructions

1. Study the data for the purpose of deciding how many cycles will be needed on the ratio scale to show these data properly.

2. Use the ratio paper found at the rear of this Manual, scale the grid and plot the two series. Number your chart, Chart 6.

3. Make a rough sketch showing what these data would look like if plotted on an arithmetic scale with a single, unbroken vertical scale. Explain why arithmetic scaling is unsatisfactory in such cases as these.

Questions

1. Is gasoline consumption increasing at a more rapid rate in Illinois than in the United States as a whole? If so, during what periods and how do you know as a result of visual analysis of these data?

PROBLEM 44. (Continued)

2. Are either of these series approximating a constant rate of increase during the period 1925–1948? How can you tell by visual analysis?

3. Make a visual analysis and then write a brief paragraph explaining how you think gasoline consumption in the United States has changed since 1945, at a constant, increasing, or decreasing rate of increase? Same for the Illinois consumption.

4. Which of the above questions could *not* be answered had the data been plotted on arithmetic instead of semi-logarithmic scale? Explain.

PROBLEM 45. REVIEW OF GRAPHICS

What kind of chart would you use to show:

1. Sales of a Harrisburg retail outlet of Montgomery Ward and Company compared to the total sales of Montgomery Ward by months, 1945–1950.

2. Cost of living for a 4-person manual worker's family at maintenance level in Chicago, broken down into food, clothing, shelter, and fuel and light components, one estimate for each year, 1940–1945.

3. The total number of shares traded, New York Stock Exchange, by months, 1947–1950.

4. Income per capita in the states of Illinois, Michigan, Indiana, and Ohio.

Questions on Construction of Charts

1. Must a properly drawn statistical chart always show zero on its scale?

2. When should a bar chart be used in preference to a line chart?

3. When is a subdivided bar chart more desirable than a simple bar chart?

4. When should a ratio scale be used instead of a natural number scale?

5. Should historical data be plotted to the beginning, middle, or end of the time unit to which it refers?

SECTION V

The Presentation of Statistical Data

1. Construction of a Frequency Table (Chapter VIII of text) Problems 46, 46A
2. Graphic Presentation, Simple Frequency Distributions (Chapter VIII of text) Problems 47, 47A
3. The Ogive (Chapter VIII of text) Problems 48, 49, 50, 51

THE PRESENTATION OF STATISTICAL DATA

Frequency Distributions and Their Graphic Presentation

PROBLEM 46. CLASSIFICATION OF QUANTITATIVE DATA BY MAGNITUDE

In Problem 28, you selected four random samples from Tables 10 and 11 in which were recorded the cost of meals served at a University Commons on December 11, 1941 and March 9, 1950. You were asked to save copies of your samples so that those results might be used again without duplicating the effort of drawing additional simple random samples. Now we wish to undertake a condensation of these data by organizing them in the form of a frequency distribution. Our problem is to accomplish this without the loss of essential detail.

Instructions

1. Consider the two large samples which you drew at random, one from Table 10 and one from Table 11. Examine the data for a round number tendency, for a tendency to group at certain values. Consider the range of the variation in each series and the number of items in each sample. When these characteristics of the data have been examined, decide on the class interval to be used for each series, and select the class limits which promise to give satisfactory classifications. The formula shown on page 218 of the text may be used as a rough guide to the class interval but the restricted utility of this formula must be kept in mind.

2. When you have decided, tentatively, on the classification system or systems to be used, turn to Forms 1 and 2. Enter your class limits for the 1941 series in the left-hand column of Form 1, for the 1950 series in the left-hand column of Form 2. Then tally the number of values in each classification and summate to secure the number of items in each class. Check the total frequencies against the number of items in your samples.

3. Double the size of the class interval by combining the frequencies as suggested in the last column to the right of Forms 1 and 2.

4. If you are dissatisfied with the results secured, begin over again with a new and better classification system and complete the tabulation for the new class intervals as before.

5. Make a copy of your two frequency distributions for use in the next problem.

Questions

1. Are your stated class limits the actual ones? If not, what are the actual class limits and why did you select the particular ones you have employed? Are your data discrete or continuous, and did this fact enter into your selection of class limits? Explain.

2. Do you consider your class interval satisfactory as originally written? As doubled? Give your reasons in each case.

3. What are the mid-points provided by your classification system? Justify their use.

4. Do you have any evidence in these distributions that the data are or are not homogeneous? Explain.

5. How does the frequency distribution provide a more satisfactory arrangement of the data than an array? How a less satisfactory one?

PROBLEM 46. (Continued)

FORM 1

FREQUENCY DISTRIBUTION ANALYSIS COST OF LUNCHEON, UNIVERSITY COMMONS
DECEMBER 11, 1941

Cost in Cents	*Tally*	*Number in Each Class (Frequencies)*	*Frequencies with Enlarged Class Interval*
Total			

PROBLEM 46. (Continued)

FORM 2

FREQUENCY DISTRIBUTION ANALYSIS COST OF LUNCHEON, UNIVERSITY COMMONS
MARCH 9, 1950

Cost in Cents	*Tally*	*Number in Each Class (Frequencies)*	*Frequencies with Enlarged Class Interval*
Total			

PROBLEM 46A. CLASSIFICATION OF QUANTITATIVE DATA BY MAGNITUDE

If, instead of Problem 28, you completed Problem 29 dealing with sampling, complete the operations required under Problem 46 with those data.

PROBLEM 47. CHARTING THE FREQUENCY DISTRIBUTION

Having secured the frequency with which each value appears in the quantitative classifications, Problem 46 or 46A, on Forms 1 and 2, we must now consider methods of presenting these results graphically.

Instructions

1. Construct histograms, one each for the two sets of data with which you worked in Problem 46. Number your charts, Chart 7 and 8.
2. Smooth both of these histograms.
3. Construct polygons, one each for the two sets of data from Problem 46. Number your charts, Chart 9 and 10.

Questions

1. Which type of chart, histogram, or polygon, *should be used* in charting these data? Explain.
2. What is the justification for smoothing a histogram and what are the rules which guided you in your smoothing?
3. In constructing the polygon, are the values of the frequencies plotted to the lower limit of the class, the upper limit, or the mid-point? Explain.
4. If the data charted are continuous values selected at random from a parent population, the histogram, the smoothed histogram or the polygon may be used. Which do you recommend and what is the basis of your recommendation?
5. Can you tell by inspection which amount appears to be spent for luncheon more frequently than any other amount in 1941 and 1950?
6. Can you tell by inspection of your charts whether there is more variation in the amounts spent in 1941 than in 1950? Explain.

PROBLEM 47A. CHARTING THE FREQUENCY DISTRIBUTION

If, instead of Problem 46, you worked Problem 46A, complete the operations required under Problem 47 for the kilowatt-hour data with necessary modifications.

The Ogive

PROBLEM 48. CUMULATIVE FREQUENCY DISTRIBUTION, THE OGIVE

In Table 22 are shown hourly (straight-time) wage rates for sewing machine operators in the overalls and industrial garments industry for all plants employing 21 or more employees on the border of Kentucky and West Virginia (13 plants), September 1949.

PROBLEM 48. (Continued)

TABLE 22

HOURLY WAGE RATES FOR SEWING MACHINE OPERATORS, INDUSTRIAL GARMENTS INDUSTRY, KENTUCKY-WEST VIRGINIA BORDER AREA
SEPTEMBER 1949

Hourly Wage Rates	*Workers*		*Cumulated Frequencies*			
			Number of Workers		*Percentages*	
	Number	*Per Cent*	*Less than*	*More than*	*Less than*	*More than*
$ .40 and under $ .50	7	.85				
.50 and under .60	38	4.58				
.60 and under .70	146	17.61				
.70 and under .80	149	17.97				
.80 and under .90	215	25.94				
.90 and under 1.00	126	15.20				
1.00 and under 1.10	66	7.96				
1.10 and under 1.20	39	4.70				
1.20 and under 1.30	28	3.38				
1.30 and under 1.40	9	1.09				
1.40 and under 1.50	6	.72				
Total	829	100.00				

Source: U. S. Department of Labor, Hourly Wage Rates Report No. 7299, September 1949.

Instructions

1. Cumulate the frequencies on a "more than" and a "less than" basis.
2. Cumulate the percentage frequencies on a "more than" and "less than" basis.
3. Plot the cumulated frequencies calculated in 1 on a "more than-less than" chart. Number your chart, Chart 11.
4. Plot the cumulated frequencies calculated in 2 on a "more than-less than" chart. Number your chart, Chart 12.

Questions

1. How did you plot the "more than" series, i.e., to the upper limit of the class, the mid-point, or the lower limit? Explain.
2. Answer the same question for the "less than" series.
3. What is the lowest hourly wage rate received by the highest 50 per cent of the workers? The highest wage received by the lowest 40 per cent? (Interpolate from your chart.)
4. Recent legislation by the U. S. Congress raised minimum wages to 75 cents per hour for products moving into interstate commerce. How many workers in these plants would have their wages increased by such legislation? Same question, but what percentage of the workers? (Read from chart.)
5. What is the highest wage received by the lowest one-third of the workers? (Use chart only for answer.)

PROBLEM 49. CUMULATIVE FREQUENCY DISTRIBUTION

In the Federal Reserve Bulletin of June 1949 will be found partial results of the Board of Governors' Survey of Consumer Finances in 1949. The data of Table 23 come from this source.

PROBLEM 49. (Continued)

TABLE 23

DISTRIBUTION OF SPENDING UNITS BY INCOME GROUPS, 1948 AND 1946
UNITED STATES

Annual Money Income before Taxes	*Spending Units*	
	1948 Per Cent	*1946 Per Cent*
Under $1,000	12	17
$1,000–$1,999	18	23
2,000– 2,999	23	25
3,000– 3,999	20	17
4,000– 4,999	12	8
5,000– 7,499	10	6
7,500 and over	5	4
Total	100	100
Median income	$2,840	$2,300

Source: *Federal Reserve Bulletin*, July 1949, Board of Governors, Federal Reserve System, Washington, p. 780.

Instructions

1. Cumulate the frequencies on "more than" or "less than" basis, or both of these if necessary to answer the questions which follow.
2. Plot the cumulative series. (Notice that class intervals are not uniform.) Number your chart, Chart 13.
3. Determine the value of the medians by interpolation on the charts.

Questions

1. Is there evidence that the income has become more equally distributed among spending units in 1948 contrasted to 1946? What is your evidence?
2. What per cent of the spending units had $4,000 or more in 1946? in 1948?
3. What per cent had less than $2,500 in 1946? in 1948?
4. What was the lowest income received by the highest one-third in 1946? in 1948?
5. You have two independent estimates of the median value for each series; one is given under the table, and another is found by interpolation on the Chart. Do they agree in value? Explain.

PROBLEM 50. THE CUMULATIVE FREQUENCY DISTRIBUTION

The data shown in Chart I were used by the Packard Motor Company in an advertisement which emphasized the gasoline economy of a new Packard Eight with overdrive. The data from the chart are reproduced in Table 24 below.

PROBLEM 50. (Continued)

CHART 1

ROAD MILES PER GALLON
135-HP PACKARD EIGHT WITH OVERDRIVE

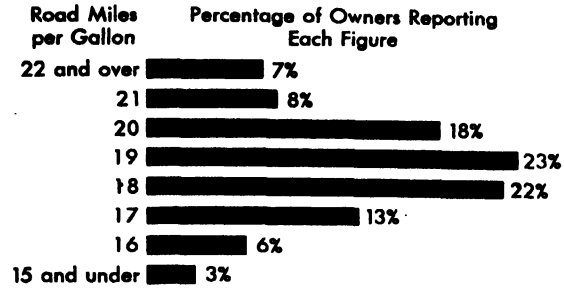


TABLE 24

ROAD MILES PER GALLON
135-HP PACKARD EIGHT WITH OVERDRIVE

Miles per Gallons	*Per Cent of Owners*
22 and over	7
21	8
20	18
19	23
18	22
17	13
16	6
15 and under	3
Total	100

Instructions

1. Assume that miles per gallon are reported at mid-points of class intervals, except for the end classes, and make "more than" and "less than" accumulations.
2. Plot the cumulative frequency distribution. Number your chart, Chart 14.

Questions

1. What per cent of the owners may expect to average 18 miles or more per gallon? 20 miles or more per gallon?
2. What is the range of mileages which will include the middle 50% of owners?
3. What is the average mileage which will be exceeded by 50% of the owners.
4. Do your results check with the statement, "33% of the owners report *more* than 19 miles"?

PROBLEM 51. THE CUMULATIVE FREQUENCY DISTRIBUTION

A sample of 22,000 urban and rural-nonfarm households was taken to determine the distribution of income in 1946 among these groups. Data showing the distribution of urban families and individuals by total money income received will be found in the following table.

PROBLEM 51. (Continued)

TABLE 25

DISTRIBUTION OF TOTAL MONEY INCOME
NONFARM FAMILIES AND INDIVIDUALS
UNITED STATES, 1946

Dollars	*Per Cent*
Under $500	6.4
$ 500 to 999	7.4
1,000 to 1,499	8.4
1,500 to 1,999	9.6
2,000 to 2,499	12.2
2,500 to 2,999	11.0
3,000 to 3,499	10.8
3,500 to 3,999	7.5
4,000 to 4,499	5.9
4,500 to 4,999	4.6
5,000 to 5,999	6.3
6,000 to 9,999	7.7
10,000 and over	2.1
Total	100

Median = 2,774

Based on sample of 22,000 urban and rural-nonfarm households.

Source: Department of Commerce, Bureau of the Census, *Income of Nonfarm Families and Individuals, 1946*, Series P-60 No. 1, SA — p. 285.

Instructions

1. Cumulate the frequencies on a "more than" and "less than" basis.
2. Plot the cumulated frequencies. Take care to plot the values to the proper limits of their classes. Number your chart, Chart 15.

Questions

1. Locate the median graphically from your chart. What is its value?
2. Does the median value found in 1 above agree with the median reported under Table 25, that is, $2,774? Should the value read from the chart agree with the value reported by the Bureau of the Census? Explain. If they do not agree, what explanation can you give for the difference?
3. What per cent of the families and individuals received a total money income of less than $2,000 a year? more than $2,000?
4. Do you think the method used in this study to measure income, i.e., "total money income," is a correct way to measure it? Explain.
5. Within what total money income range did the middle 50 per cent lie? the middle 80 per cent?
6. After study of this distribution, give a definition of a "high" income and state what per cent received this income or more.

SECTION VI

The Analysis of the Frequency Distribution

1. Common Averages (Chapter IX of text) and Dispersion (Chapter X of text) Problems 52, 53
2. The Geometric Mean (Chapter IX of text) Problem 54
3. The Harmonic Mean (Chapter IX of text) Problem 55
4. Summary and Review, Problem 56

PROBLEM 52. ANALYSIS OF THE FREQUENCY DISTRIBUTION

In preparation for a hearing before the regional office of the National War Labor Board, statisticians employed by the California Packing Company gathered the data shown in Table 26. The company contended that an increase in wages was necessary to retain experienced personnel in the plant. Part of the analysis was based on these data from Plant 110.

TABLE 26

WAGE RATES PER HOUR PLANT AND FIELD EMPLOYEES
CALIFORNIA PACKING COMPANY PLANT 110, 1944

Wage Rates per Hour in Cents	*Number of Employees*
60.0 and under 65.0	95
65.0 and under 70.0	144
70.0 and under 75.0	189
75.0 and under 80.0	109
80.0 and under 85.0	95
85.0 and under 90.0	73
90.0 and under 95.0	47
95.0 and under 100.0	30
100.0 and under 105.0	29
105.0 and under 110.0	3
Total	814

Instructions

1. Plot the frequency distribution of wage rates per hour for employees of Plant 110 of the California Packing Company. Number your chart, Chart 16.
2. Compute the following measures of central tendency:
   a. Arithmetic mean.
   b. Median.
   c. Mode.
3. Compute the standard deviation.
4. Compute the coefficient of variation.
5. Compute the coefficient of skewness.
6. Indicate graphically on your chart the location of each of the averages.
7. Indicate graphically on your chart the range of the mean ± 1 standard deviation.

Questions

1. (*a*) Under what circumstances will the values of the mean, median, and mode be identical? (*b*) How do they differ in this problem; that is, which is larger, which is smaller, and which comes "in between"?
2. Which average would you argue is the best to use in a wage dispute? Why?
3. Which average would you use to estimate the total weekly payroll for the plant, assuming a standard work week? Why?

PROBLEM 52. (Continued)

4. Could you use any of the averages computed to estimate total weekly payments of Social Security tax?

5. How is the distribution of hourly wage rates skewed?

6. Would an average of hourly wage rates and hourly earnings be identical in such a plant? If not, why would they differ?

7. Write a paragraph explaining the meaning of the standard deviation in this analysis.

8. For comparison with wage distributions in other plants operated by this company, what measure of dispersion should be used and why?

PROBLEM 53. ANALYSIS OF A FREQUENCY DISTRIBUTION

Sales Management magazine in its May issue each year reports data useful for the analysis of markets in the counties and principal cities of the United States. Table 27 shows a frequency distribution of effective buying income per capita for Indiana counties, 1948, from this source.

TABLE 27

ESTIMATES OF EFFECTIVE BUYING INCOME PER CAPITA, INDIANA COUNTIES, 1948

E.B.I. per Capita	*Number of Counties*
$ 600 and under $ 700	4
700 and under 800	10
800 and under 900	13
900 and under 1,000	11
1,000 and under 1,100	14
1,100 and under 1,200	13
1,200 and under 1,300	9
1,300 and under 1,400	6
1,400 and under 1,500	6
Over 1,500	3
Total	89

Source: *Sales Management* Survey of Buying Power, May 10, 1949, pp. 326–336. Further reproduction not licensed.

Instructions

1. Plot the above data. Number your chart, Chart 17.
2. Calculate the value of the following measures:
   - *a.* Median
   - *b.* First quartile
   - *c.* Third quartile
   - *d.* Quartile deviation
   - *e.* K
   - *f.* Coefficient of dispersion
3. Indicate graphically on the chart values computed under instructions *a*, *b*, and *c*.

PROBLEM 53. (Continued)

Questions

1. Could you compute an arithmetic mean from the data on effective buying income per capita for Indiana counties as shown in Table 27? Explain.
2. The value for the three counties in the "open-end" class are $1,588, $1,800, and $1,825. With this information available, could you compute an arithmetic mean of per capita income by counties? If so, explain the meaning of such average.
3. What is the range within which the middle 50 per cent of the counties lie?
4. Under what conditions will "K" equal the median?
5. Write a paragraph explaining the uses of effective buying power estimates to a sales manager and the meaning of the statistical measures you have computed as they relate to these data.

PROBLEM 54. THE GEOMETRIC AVERAGE

In Table 28 will be found listed 24 commodities imported into this country with their 1939 and 1947 prices, the ratio of 1947 prices to 1939 prices and the logarithms of these ratios excepting for a few values which the student is expected to fill in.

TABLE 28

PRICES OF 24 COMMODITIES IMPORTED INTO THE UNITED STATES, 1939–1947

Commodity	*Prices*		*Price Relative 1939 = 100*	*Log of Price Relative*
	1939	*1947*		
Cheese, per lb.	$0.217	$ 0.539	248.4	2.3952
Wheat, per bu.	0.56	1.45	258.9	2.4131
Bananas, per bunch	0.5091	0.825	162.1	2.2098
Cocoa, per lb.	0.042	0.255	607.1	2.7833
Coffee, per lb.	0.069	0.241	349.3	2.5432
Tea, per lb.	0.216	0.407	188.4	2.2751
Cane sugar, per lb.	0.0215	0.0492	228.8	2.3594
Rubber, crude, per lb.	0.16	0.201	125.6	2.0988
Shellac, per lb.	0.08	0.487	608.7	2.7844
Copra, per lb.	0.016	0.079	493.7	2.6934
Flaxseed, per bu.	1.15	6.75	586.9	2.7685
Jute burlap, per lb.	0.063	0.201	319.0	2.5038
Sisal, per ton	72.90	269.33	369.9	2.5681
Manila, per ton	90.83	377.35	415.4	2.6184
Silk, raw, per lb.	2.34	6.39	273.1	2.4364
Pulpwood, per cord	8.34	16.67		
Newsprint, paper, per lb.	0.022	0.044		
Petroleum, crude, per gal.	0.016	0.039		
Manganese ore, per lb.	0.0119	0.0170		
Nickel, pigs, ingots, bars, per lb.	0.2509	0.3013	120.1	2.0796
Tin, bars, blocks, pigs, per lb.	0.4495	0.7653	170.3	2.2312
Sodium nitrate, per short ton	18.55	27.23	146.8	2.1663
Fish, cured, per 200-lb. barrel				
Herring	9.30	20.35	218.8	2.3400
Mackerel	9.45	31.60	334.4	2.5243

Source: Annual Average Unit Values of Important Articles Imported (to the U. S.) *Statistical Abstract*, 1949, pp. 314–315. (U. S. Government Printing Office, Washington.)

PROBLEM 54. *(Continued)*

Instructions

1. Complete the calculation of ratios and look up the logarithms of the ratios calculated. Enter the values in the space provided in Table 28.
2. Compute the arithmetic mean of the ratios.
3. Compute the logarithmic mean of the ratios.

Questions

1. Discuss the merits of the two means computed under 2 and 3. Explain which of the means should be used to describe the average increase in the price of imported goods, 1939–1947.
2. In general, when should the geometric mean be used rather than the arithmetic mean?
3. Would either of the averages you have computed serve reasonably well as an index of imported goods prices? What questions would you want to have answered before committing yourself to a reply?

PROBLEM 55. HARMONIC MEAN — TIME RATES

A chain of garages in Chicago proposed to offer a standard price for the labor required for a motor overhaul job on a Chevrolet car. The time required to do the job was the key to labor cost and an average time was needed. A time study of 50 jobs in their shops yielded the following data, which have been organized into a frequency distribution in Table 30.

TABLE 29

MANHOURS REQUIRED TO COMPLETE MOTOR OVERHAUL
FIFTY OBSERVATIONS
(In hours and hundredths of hours)

8.40	8.30	7.75	7.65	7.70
7.80	7.95	8.35	8.15	8.45
8.60	7.60	7.75	9.15	8.35
7.95	8.10	7.90	8.75	7.95
8.55	8.10	8.24	8.50	7.95
8.06	7.79	7.55	8.20	8.20
8.15	8.40	8.75	8.55	7.50
7.75	8.95	8.20	8.10	8.15
7.89	8.30	8.35	7.98	8.35
7.85	7.66	8.15	8.15	7.90

TABLE 30

MANHOURS REQUIRED TO COMPLETE MOTOR OVERHAUL
FIFTY OBSERVATIONS

Hours	*No. of Overhauls*
7.45–7.649	3
7.65–7.849	8
7.85–8.049	9
8.05–8.249	13
8.25–8.449	8
8.45–8.649	5
8.65–8.849	2
8.85–9.049	1
9.05–9.249	1
Total	50

PROBLEM 55. *(Continued)*

The arithmetic mean is 8.146 hours, the mode is 8.138 hours, and the geometric mean is 8.138 hours. The standard deviation is 0.352 hour, the average deviation is .271 hour, and the quartile deviation is 0.234 hour.

Instructions

1. Plot the data shown in the frequency distribution. Number your chart, Chart 18.
2. Compute the harmonic mean.
3. Show the harmonic mean and arithmetic mean values on the chart.

Questions

1. Which is smaller, the arithmetic mean or harmonic mean?
2. Explain carefully how using the harmonic mean weights the items.
3. Argue that the harmonic mean of these time study data is the proper average to use in estimating average number of jobs per week in these shops.
4. If shop rates are $5 per hour, what would be your best estimate of a standard labor cost for the job?

PROBLEM 56. REVIEW OF AVERAGES AND DISPERSION

PART I

Averages and Dispersion Computed from an Array

The following data represent three random samples of 20 items each drawn from a meter reader's record of electricity consumed by householders in a particular community. The readings are in kilowatt-hours.

Sample a	*Sample b*	*Sample c*
16	10	18
60	2	36
24	30	8
36	16	4
12	60	62
22	54	18
22	34	60
12	24	78
16	16	10
28	32	16
58	12	4
36	52	12
20	74	36
24	42	26
56	34	12
28	34	2
28	2	14
28	24	20
6	20	78
26	14	34

PROBLEM 56. (Continued)

Instructions

1. Arrange the items in one of the above samples as an array.
2. Determine the arithmetic mean, the median, and the mode from the array. (Indicate the mode by selecting the most frequent value.)
3. Compute the standard deviation from the array.

Questions

1. What difficulties did you encounter in locating the mode from the array? Did you encounter the same problems in computing the arithmetic mean? the median?
2. How may one determine the direction of skewness from an array? Is the method satisfactory?

PART II

1. Assuming variation among the values of a series, the geometric mean is never larger than the arithmetic mean but is usually smaller. Explain why such is the case.
2. Under what conditions is the harmonic mean the most satisfactory measure of central tendency? Explain with an illustration.
3. What kind of an average (mode, arithmetic mean, median, geometric, or harmonic) do you consider the most meaningful in the following cases? Give your reasons.
   a. The average number of axles produced per lathe-hour. If your answer depends on how the original data are quoted, explain.
   b. The average percentage increase in future prices for grains on the Chicago Board of Trade, August 1948 to December 1949.
   c. To estimate collections under a social security tax when numbers of workers in each weekly wage class is known.
4. How is a coefficient of dispersion or variation more significant than the absolute dispersion in miles in comparing the performances of several grades of automobile tires?

SECTION VII

The Interpretation of Sampling Statistics

1. Dispersion and Simple tests of Hypotheses (Chapter X of text), Problems 57, 58, 59
2. Significances of Differences — Two Sample Means (Chapter X of text), Problems 60, 61

THE INTERPRETATION OF SAMPLING STATISTICS

Standard Error of a Mean and Tests of Hypotheses

PROBLEM 57. DISPERSION AND TESTS OF HYPOTHESES

The automatic loading machine at the Climax Cement Plant is designed to fill a bag with 100 pounds of cement. It is known that no machine is perfect, and weight tests are frequently conducted because overloading cuts profits and underloading leads to complaints by customers.

One such test involved choosing 65 bags and weighing each of them. Their mean was 102 pounds, and their standard deviation, 4 pounds.

Questions

1. May we conclude that about one-sixth (about 16%) of the bags loaded by that machine have been leaving the plant with not over 98 pounds of cement each?
2. For this particular sample ($N=65$), what is the standard error of the mean?
3. The machine is designed to give 100-pound loads; the sample indicates a mean of 102 pounds. Compute the value of T (critical ratio) in this situation and for this sample.
4. When allowances are made for random errors of sampling, may we conclude that the machine *is* overloading, because the difference between 102 pounds and 100 pounds cannot easily be attributed to such sampling errors? Explain.

PROBLEM 58. DISPERSION AND TESTS OF HYPOTHESES

The Metropolitan Rapid Transit Company assembles its passenger trains for each run on the basis of a forecast of the traffic the trains will be required to haul each particular trip. Ideally, train size would be adjusted so that vacant seats are few but no passenger is required to stand.

As a result of complaints of inadequate service, a study was made between two points of heavy traffic. A count was made of vacant seats (a vacant seat was counted as a plus 1) and passengers standing (a standee was counted as a minus 1) on 100 trips, chosen by random methods. The work sheets looked like this:

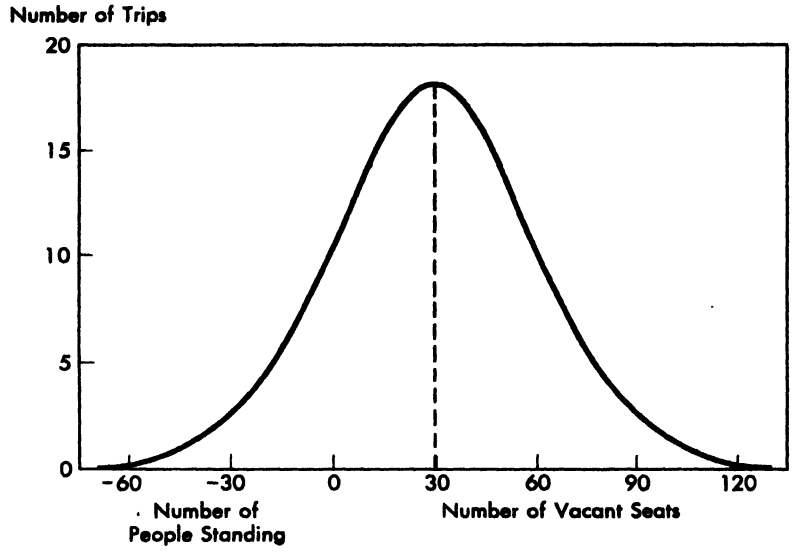
	Standing (−)	*Vacant Seats* (+)	*Total*
Trip 1	5	10	+ 5
Trip 2	0	30	+30
Trip 3	25	0	−25
Trip 4	0	0	0
etc.			

A frequency distribution was then constructed of the totals for each of the 100 trips which, when smoothed, was as follows (Exhibit 3):

PROBLEM 58. (Continued)

EXHIBIT 3

SMOOTHED DISTRIBUTION OF TRIPS BY NUMBER OF "STANDEES" AND VACANT SEATS



The mean of this distribution is, as shown on the diagram, plus 30 (meaning that there were 30 vacant seats on the typical trip). The standard deviation is also 30.

Questions

1. On how many trips in a hundred would you expect to find no vacant seats?
2. The company contended, before the Transit Board, that sampling errors were important in this case and that forecasts were made to provide a forty-seat margin for unforeseen loads. Can the difference between 40 and the mean 30 of the sample be explained by random errors of sampling? (Compute T and show whether you agree.)

PROBLEM 59. TESTING AN HYPOTHESIS

As a jobber of photographic equipment and supplies, Acme sells a 500-watt bulb for motion picture projectors. The manufacturer of the bulb guarantees 200 hours of use under "normal" conditions.

Acme has received numerous complaints and requests for adjustments through retail outlets. As a consequence, 50 bulbs are tested under "normal" conditions. The average life of the 50 bulbs proved to be 180 hours: standard deviation, 70 hours.

Instructions

1. Compute the standard error of the mean.
2. Compute the T-value.
3. Write a paragraph explaining how T may be used in testing an hypothesis.

Questions

1. Would you say that the bulbs are reasonably uniform as to quality? Explain your answer.
2. Buyers of the bulb will not, ordinarily, complain if the bulb lasts 100 hours or more. Accord-

PROBLEM 59. (Continued)

ing to this test about how many bulbs sold, in a 100, may lead to complaints and requests for adjustments? Explain your answer.

3. Does this test indicate, beyond all reasonable doubt, that Acme's supplier (the manufacturer) is furnishing bulbs which are not up to the guarantee of 200 hours? Explain.

4. At what point (number of service hours) would you refuse to accept further deliveries of bulbs from this supplier under the 200-hour guarantee? Explain fully.

5. What question would you ask concerning the sample before accepting the results given above?

PROBLEM 60. SIGNIFICANCE OF DIFFERENCE BETWEEN MEANS OF CHECKS PAID LUNCHEON HOUR, A LA CARTE DIVISION, UNIVERSITY COMMONS, DECEMBER 11, 1941 AND MARCH 9, 1950

In this problem we propose to find out whether there is a significant difference in the average check paid at a University Commons at the luncheon hour as measured by samples of data for December 11, 1941 and March 9, 1950.

Samples of 100 each were drawn at random from the checks for the two luncheon hours (see Tables 10 and 11). The means, standard deviations and standard errors of the means are shown below in Table 31.

TABLE 31

100 ITEM SAMPLES DRAWN AT RANDOM FROM LUNCHEON CHECKS PAID, DECEMBER 11, 1941 AND MARCH 9, 1950

December 11, 1941		*March 9, 1950*	
Cost of Meals in Cents	*Number of Meals*	*Cost of Meals in Cents*	*Number of Meals*
11–15	6	13–19	1
16–20	3	20–26	2
21–25	14	27–33	2
26–30	19	34–40	2
31–35	13	41–47	5
36–40	12	48–54	13
41–45	10	55–61	19
46–50	14	62–68	28
51–55	3	69–75	20
56–60	4	76–82	5
61–65	2	83–89	3
Totals	100		100
Mean	34.9		60.9
σ_S	11.1		13.1
σ_M	1.1		1.3

Instructions

1. Compute the standard error of the difference of the two means.

2. Compute the critical ratio for the difference between the two means.

3. Turn to a table of areas of the normal curve and estimate the probabilities of a difference so large or larger occurring as a result of chance.

PROBLEM 60. (Continued)

Questions

1. Are there significant differences between the means of these samples? Explain carefully.
2. Is the hypothesis that the two samples come from populations with the same mean value a tenable hypothesis? Explain.

PROBLEM 61. THE SIGNIFICANCE OF THE DIFFERENCE BETWEEN MEANS

The article in *Life* magazine referred to in Problem 14 drew some definite conclusions concerning the comparative earnings of Phi Beta Kappas, Varsity letter men and those "scholar-athletes" who were in both groups. "Contrasting the grind, or honors' man, with the athlete reveals the uselessness of muscles in the outer world," it was said. Calculations showed that the class of '32 had 31 Phi Beta Kappa men, whose average earnings were, in 1947, $10,517. The 48 varsity letter men earned only $8,260 on that date. But the five aristocrats of the class, the scholar-athletes who won both a Phi Beta Kappa key and a varsity letter, had done the best of all. Their earnings averaged exactly $11,000 per year in 1947.

No standard deviations were given in the article. Assume, however, that the standard deviation for the series of 31 Phi Beta Kappas was $5,000; for the 48 letter men, $4,000; and for the 5 "aristocrats," $5,500.

Questions

1. Are the conclusions reported based on a sample? If so, what is the population? What opportunities for bias do you see in the manner of selecting the group for analysis?
2. Is there a reasonable doubt that Phi Beta Kappas do better 15 years after graduation than the varsity letter men? Explain.
3. Test the hypothesis that Phi Beta Kappas come from a population which has the same mean as the population of scholar-athletes. Explain your results.
4. Record any doubts you may have concerning the conclusions reached in answering questions 2 and 3.
5. How does the fact that one group has only 5 members influence your conclusions? Explain.

SECTION VIII

Index Numbers

1. Index Numbers of Prices (Chapters XI and XII of text), Problems 62, 63, 65
2. Index Numbers of Physical Volume (Chapter XI of text), Problem 64
3. Correction of Data for Price Level Changes (Chapter XII of text), Problem 66

Construction of Index Numbers of Prices

PROBLEM 62. RETAIL PRICES OF RESIDENTIAL HEATING FUELS, CHICAGO

A property management agency responsible for the operation of a group of small rental properties in Chicago is preparing a brief to be used in support of a requested rental increase. The brief is prepared in behalf of the clients of the property management agency who are the owners of the properties.

As part of the argument, it is desired to measure the average increase in the prices of fuel furnished by the owners.

The task of collecting the data and computing the average changes in fuel prices, 1941–1949, is assigned to a junior executive in the purchasing department of the agency. The types of fuels commonly furnished and their December retail prices in the Chicago market are collected and shown in Table 33.

TABLE 33

RESIDENTIAL HEATING FUELS, AVERAGE DECEMBER RETAIL PRICES, CHICAGO, 1941–1949

Year	*Bituminous Coal and Coke Prices*				*Fuel Oil Prices*	
	Low & Medium Volatile Nut $/ton	*Eastern High Volatile Stoker* $/ton	*Western High Volatile Stoker* $/ton	*Coke Nut* $/ton	*Range Oil* $/gal.	*No. 2* $/gal.
1941	10.21	9.24	7.49	13.81	.0918	.0765
1942	10.26	9.37	7.56	13.84	.0918	.0796
1943	10.73	9.91	8.16	14.39	.0949	.0796
1944	10.94	9.99	8.30	15.09	.0796	.0745
1945	11.24	10.22	8.45	15.49	.0796	.0745
1946	12.56	11.66	9.94	17.62	.0969	.0887
1947	15.64	15.84	12.69	20.93	.1514	.1316
1948	17.74	17.84	14.27	21.99	.1540	.1357
1949	18.24	18.29	15.02	22.49	.1320	.1193

Source: Adapted from: *Residential Heating Fuels, Retail Price* 1941–1948 Bureau of Labor Statistics, U. S. Department of Labor, Bulletin No. 950, p. 15; 1949 Values from: *Retail Prices of Residential Heating Fuels*, by cities, December 15, 1949, Preliminary Report, Bureau of Labor Statistics, U. S. Department of Labor.

PART I

Type of Index—The Simple Aggregative of Actual Prices, 1941-Base

As a first approach to the problem, the person to whom the analysis has been assigned simply adds up the per ton and per gallon prices for each year 1941–1948 as shown in Table 34. He then converts the totals of unit prices to relatives of their 1941 total. This is shown in the last column of Table 34.

TABLE 34

SIMPLE AGGREGATIVE OF ACTUAL FUEL PRICES, CHICAGO
1941 = 100

Year	*Sum of Prices*	*Index*
1941	$40.92	100.0
1942	42.20	103.1
1943	43.36	106.0
1944	44.47	108.7
1945	45.55	111.3
1946	51.97	127.0
1947	64.48	157.6
1948	72.13	176.3
1949		

Source: Based on Table 33.

Instructions

1. Complete the calculations for the year 1949.
2. Assume you are employed as a consultant at this point. Prepare a statement designed to:
   a. Show the junior executive whether or not the items are equally weighted in his calculation.
   b. Show him what the weighting system really is.
   c. Explain how the index would change if fuel oil were priced in 42-gallon barrels instead of gallons as shown in Table 33.
   d. Tell him what question his calculation answers and consider whether these results provide an adequate answer to the problem before the agency.

PART II

Type of Index—Simple Average of Relatives and Geometric Average of Relatives

Although impressed by the weaknesses of the first method tried, the agency continues to insist on simplicity. It is anticipated that the case may have to go to court and, in the opinion of the agency, complicated formulas should be avoided.

As consultant, you are aware of two other simple methods of averaging prices—the simple arithmetic average of relatives and the simple geometric average of relatives. In Table 35 are the price relatives (1941 base) and their averages and in Table 36 are the logarithms of these relatives, their averages and antilogs.

TABLE 35

SIMPLE AVERAGE OF RELATIVES METHOD, CALCULATION OF FUEL PRICE INDEX, CHICAGO, 1941 = 100

Type of Fuel	*1941*	*1942*	*1943*	*1944*	*1945*	*1946*	*1947*	*1948*	*1949*
Bituminous coal									
Nut	100.0	100.5	105.1	107.1	110.1	123.0	153.2	173.8	
Eastern stoker	100.0	101.4	107.3	108.1	110.6	126.2	171.4	193.1	
Western stoker	100.0	100.9	108.9	110.8	112.8	132.7	169.4	190.5	
Coke—nut	100.0	100.2	104.2	109.3	112.2	127.6	151.6	159.2	
Range oil	100.0	100.0	103.4	86.7	86.7	105.6	164.9	167.8	
No. 2 fuel oil	100.0	104.1	104.1	97.4	97.4	116.2	172.0	177.4	
Total	600.0	607.1	633.0	619.4	629.8	731.3	982.5	1061.8	
Index	100.0	101.2	105.5	103.2	105.0	121.9	163.8	177.0	

Source: Based on Table 33.

PROBLEM 62. (Continued)

TABLE 36

GEOMETRIC AVERAGE OF RELATIVES METHOD, CALCULATION OF FUEL PRICE INDEX, CHICAGO, 1941 = 100

Logarithms of Fuel Price Relatives

Type of Fuel	*1941*	*1942*	*1943*	*1944*	*1945*	*1946*	*1947*	*1948*	*1949*
Bituminous coal									
Nut	2.0000	2.0022	2.0216	2.0298	2.0418	2.0899	2.1853	2.2400	
Eastern stoker	2.0000	2.0060	2.0306	2.0338	2.0438	2.1016	2.2340	2.2858	
Western stoker	2.0000	2.0039	2.0370	2.0445	2.0523	2.1229	2.2289	2.2799	
Coke — Nut	2.0000	2.0007	2.0179	2.0386	2.0500	2.1059	2.1807	2.2019	
Range oil	2.0000	2.0000	2.0145	1.9380	1.9380	2.0237	2.2172	2.2248	
No. 2 fuel oil	2.0000	2.0175	2.0175	1.9886	1.9886	2.0552	2.2355	2.2480	
Total	12.0000	12.0303	12.1391	12.0733	12.1145	12.4992	13.2816	13.4804	
Log M_g	2.0000	2.0051	2.0232	2.0122	2.0191	2.0832	2.2136	2.2467	
Index	100	101.2	105.5	102.9	104.5	121.1	163.5	176.5	

Source: Based on Table 35.

Instructions

1. Complete the calculations in Tables 35 and 36 for 1949.
2. List the defects of the simple arithmetic average of relatives.
3. Show how and to what extent the simple geometric average of relatives is free of the defects listed under 2.
4. If, because of agency insistence on an easily understood formula, you are required to recommend one of the three indexes thus far calculated which would you recommend and why?
5. In anticipation of a formal hearing, what criticisms would you expect to have levied against your recommended index and what, if any, defenses could you present? Explain.

PART III

Type of Index — Weighted Aggregative and Weighted Average of Relatives

In spite of agency insistence on a simple formula for averaging prices, you consider it your duty as consultant to show your employer the advantages of an index number formula with a realistic weight pattern. As consultant, you make the point that each price series should be given weight in the index in proportion to its contribution to the total fuel cost to the clients of the agency. Total fuel cost for any year is established by two factors: (*a*) the price per unit and (*b*) the number of units involved. The data relating to the quantities of the various fuels used in 1941 and the per unit and total cost of these fuels are presented in Table 37.

PROBLEM 62. (Continued)

TABLE 37

PRICES, QUANTITIES AND COST OF FUELS FURNISHED SELECTED SMALL CHICAGO RENTAL PROPERTIES, 1941

Type of Fuel	*Price*	*Quantity*	*Cost*	*Cost as a % of Total Fuel Cost*
Bituminous coal				
Nut	$10.21 per ton	600 tons	$ 6,126	21.0%
Eastern stoker	9.24 per ton	700 tons	6,468	22.2
Western stoker	7.49 per ton	800 tons	5,992	20.6
Coke — nut	13.81 per ton	400 tons	5,524	18.9
Range oil	.0918 per gal.	30,000 gals.	2,754	9.4
No. 2 fuel oil	.0765 per gal.	30,000 gals.	2,295	7.9
Totals			$29,159	100.0%

Source: Prices, Table 33; quantities, hypothetical.

You decide that the best method of demonstrating proper weighting in an index would be to use both the weighted aggregative index with physical quantity weights on a 1941 base, and the weighted average of relatives index. The weights in the latter form of index are the per cent that each fuel represented to total cost of fuels in the base period, 1941. In Table 37 are the various fuel prices weighted by the quantities used in 1941. In Table 38 are the price relatives, each given an importance in the index that reflects its share of fuel cost in the base period.

TABLE 38

WEIGHTED AGGREGATIVE METHOD, CALCULATION OF FUEL PRICE INDEX, CHICAGO
BASE YEAR QUANTITY WEIGHTS, 1941 = 100

Type of Fuel	*1941*	*1942*	*1943*	*1944*	*1945*	*1946*	*1947*	*1948*	*1949*
Bituminous coal									
Nut	6,126	6,156	6,438	6,564	6,744	7,536	9,384	10,644	
Eastern stoker	6,468	6,559	6,937	6,993	7,154	8,162	11,088	12,488	
Western stoker	5,992	6,048	6,528	6,640	6,760	7,952	10,152	11,416	
Coke — nut	5,524	5,536	5,756	6,036	6,196	7,048	8,372	8,796	
Range oil	2,754	2,754	2,847	2,388	2,388	2,907	4,542	4,620	
No. 2 fuel oil	2,295	2,388	2,388	2,235	2,235	2,661	3,948	4,071	
Totals	29,159	29,441	30,894	30,856	31,477	36,266	47,486	52,035	
Index	100.0	101.0	105.9	105.8	107.9	124.4	162.9	178.5	

Source: Calculated from data in Tables 33 and 37.

PROBLEM 62. (Continued)

TABLE 39

WEIGHTED AVERAGE OF RELATIVES METHOD, CALCULATION OF FUEL PRICE INDEX, CHICAGO
BASE YEAR WEIGHTS 1941 = 100

Type of Fuel	*1941*	*1942*	*1943*	*1944*	*1945*	*1946*	*1947*	*1948*	*1949*
Bituminous coal									
Nut	2,100	2,110	2,207	2,249	2,312	2,583	3,217	3,650	
Eastern stoker	2,220	2,251	2,382	2,400	2,455	2,802	3,805	4,287	
Western stoker	2,060	2,079	2,243	2,282	2,324	2,734	3,490	3,924	
Coke — nut	1,890	1,894	1,969	2,066	2,121	2,412	2,865	3,009	
Range oil	940	940	972	815	815	993	1,550	1,577	
No. 2 Fuel oil	790	822	822	769	769	918	1,359	1,401	
Totals	10,000	10,096	10,595	10,581	10,796	12,442	16,286	17,848	
Index	100.0	101.0	106.0	105.8	108.0	124.4	162.9	178.5	

Source: Calculated from data in Tables 35 and 37.

Instructions

1. Follow the instructions for the construction of a weighed aggregative index given in the text, and compute the fuel price index for 1949, Table 38.

2. Using the weighted average of relatives method, complete the fuel price index in Table 39, for 1949.

Questions

1. Are the results secured for the fuel price index for 1949 obtained by these two methods identical? Should they be identical? If not, why not? If so, why?

2. Is the weighting system in the weighted aggregative index superior to that in the simple aggregative? Why? Is it superior to the simple average of relatives index computed earlier in this problem? Why?

3. Answer the same question with respect to the superiority of the weighted average of relatives index over the simple aggregative and the simple arithmetic average of relatives.

4. Which one of the five types of index numbers computed would you recommend as being both most defensible and most suitable for agency use? Explain.

PART IV

Conclusions—Comparison of Types of Index Numbers

Table 40 contains the index numbers computed by each of the several methods discussed in this problem for the years 1941 through 1948.

PROBLEM 62. (Continued)

TABLE 40

COMPARISON OF INDICES—CHICAGO FUEL PRICES, VARIOUS METHODS OF COMPUTATION, 1941 = 100

Year	*Simple Aggregative*	*Simple Average of Relatives*	*Simple Geometric Average of Relatives*	*Weighted Aggregative*	*Weighted Average of Relatives*
1941	100.0	100.0	100.0	100.0	100.0
1942	103.1	101.2	101.2	101.0	101.0
1943	106.0	105.5	105.5	105.9	106.0
1944	108.7	103.2	102.9	105.8	105.8
1945	111.3	105.0	104.5	107.9	108.0
1946	127.0	121.9	121.1	124.4	124.4
1947	157.6	163.8	163.5	162.9	162.9
1948	176.3	177.0	176.5	178.5	178.5
1949					

Source: Tables 34, 35, 36, 38, and 39.

Instructions

Fill in the 1949 values for each of the types of index computed.

Questions

1. Is there any statistical bias in any of the index numbers recorded in Table 40? If so, which one and why?
2. Is there any such thing as a non-weighted index number? Explain.
3. What do you think of the selection of 1941 as a base year?
4. Why are three of the indices recorded in Table 40 unsatisfactory, and why are two of them better for general use?

Weighted Aggregative Price Index

PROBLEM 63. INDEX OF FOOD PRICES, MINNEAPOLIS AND PHILADELPHIA

In this food price index problem are presented the prices and weighting factors for twelve items which are important in the food budget of the usual city worker. The weights used are taken from the food budget studies reported in the Bureau of Labor Statistics Bulletin No. 927, *Workers' Budgets in the United States*. The number of items has been limited and the applicable weights simplified. An adequate index of food prices would cover a greater diversity of items including a representative group of prices for food and meals consumed away from home.

The food items and their prices collected by the Bureau of Labor Statistics for August of 1947, 1948 and 1949 in the cities of Minneapolis and Philadelphia are given in Table 41. The weighting factors are in Table 42. Table 43 gives some of the products of the price and the quantity weights.

Instructions

Calculate the index for each city for August 1949, using August 1947 = 100.

Questions

1. Can you tell in which city, according to the final index you have calculated, the budgeted food could be bought most cheaply?
2. What are the weights in this index?
3. State your opinion on whether the weights used apply equally well to the upper income suburbanite? to the farm laborer? to 1949 as well as 1947?

PROBLEM 63. (Continued)

4. Do the index numbers you have computed justify the statement, "It cost more to live in Philadelphia in August 1949 than in Minneapolis"? Explain.

5. What precisely can you say about prices in the two cities as a result of the index number calculations?

6. Is it necessary for the weighting factors and the prices of individual commodities to be in the same units (pound, quart, No. 2 can, etc.) in calculating an index by this method?

TABLE 41

RETAIL PRICES, SELECTED FOOD ITEMS, MINNEAPOLIS AND PHILADELPHIA AUGUST 1947, 1948, AND 1949

Commodity	*Quantity Priced*	*Prices in Cents*					
		August, 1947		*August, 1948*		*August, 1949*	
		Minneapolis	*Philadelphia*	*Minneapolis*	*Philadelphia*	*Minneapolis*	*Philadelphia*
		(1)	(2)	(3)	(4)	(5)	(6)
Flour, wheat	5 lb.	47.9	47.7	48.5	45.8	47.9	47.2
Bread, white	lb.	12.0	13.7	13.0	15.1	13.0	15.0
Hamburger	lb.	45.7	46.8	58.2	65.1	50.4	49.5
Pork chops	lb.	72.9	78.9	81.3	98.4	73.8	85.3
Butter	lb.	80.7	80.0	85.6	89.9	70.0	73.1
Milk, fresh delivered	qt.	18.0	19.0	21.0	21.0	16.5	20.3
Eggs, fresh	doz.	59.6	78.0	62.1	84.2	64.6	83.4
Apples	lb.	11.7	11.7	13.7	11.5	9.3	11.9*
Potatoes	15 lb.	73.4	68.0	80.2	84.7	68.0	81.9
Canned tomatoes	No. 2 can	20.2	17.3	18.2	17.0	16.9	14.6
Lard	lb.	23.6	24.6	28.5	29.3	17.6	19.3
Sugar	lb.	10.3	9.4	9.7	8.8	10.1	9.1

* From July 15, 1949 — not available for August 15, 1949.

Source: Prices, *Retail Prices of Food by Cities*, U. S. Department of Labor, Bureau of Labor Statistics, August 15 1947, pp. 30–31, 36–37; August 15, 1948, pp. 28–29, 34–35; August 15, 1949, pp. 28–29.

TABLE 42

ESTIMATED ANNUAL QUANTITIES CONSUMED, CITY FAMILIES AND SINGLE PERSONS, 1946–1947

Commodity	*Quantity Weights*	
Flour, wheat	37.26	(5-lb. bags)
Bread, white	289.0	(lb.)
Hamburger	272.7	(lb.)
Pork chops	116.9	(lb.)
Butter	79.0	(lb.)
Milk, fresh delivered	594.4	(qt.)
Eggs, fresh	85.2	(doz.)
Apples	400.2	(lb.)
Potatoes	26.067	(15-lb. bags)
Canned tomatoes	36.615	(cans)*
Lard	58.8	(lb.)
Sugar	181.7	(lb.)

* A No. 2 can contains 1.3 lb. The weight used represents 47.6 lb. of canned tomatoes, the amount designated as the budget quantity.

Source: *Workers Budgets in the United States, City Families and Single Persons*, 1946 and 1947, U. S. Department of Labor Statistics, Bulletin No. 927, p. 31.

PROBLEM 63. (Continued)

TABLE 43

CALCULATION OF WEIGHTED AGGREGATIVE PRICE INDEX, FOOD PRICES, MINNEAPOLIS AND PHILADELPHIA, MONTH OF AUGUST, 1947, 1948, AND 1949
1947 = 100

	Price × Quantity Weights					
	August 1947		*August 1948*		*August 1949*	
	Minneapolis	*Philadelphia*	*Minneapolis*	*Philadelphia*	*Minneapolis*	*Philadelphia*
Flour, wheat (5 lb.)	1784.8	1777.3	1807.1	1706.5		
Bread, white (lb.)	3468.0	3959.3	3757.0	4363.9		
Hamburger (lb.)	12462.4	12762.4	15871.1	17752.8		
Pork chops (lb.)	8522.0	9223.4	9504.0	11503.0		
Butter (lb.)	6375.3	6320.0	6762.4	7102.1		
Milk, fresh delivered (qt.)	10708.2	11303.1	12492.9	12492.9		
Eggs, fresh (doz.)	5077.9	6645.6	5290.9	7173.8		
Apples (lb.)	4682.3	4682.3	5482.7	4602.3		
Potatoes (15 lb.)	1913.3	1772.6	2090.6	2207.9		
Canned tomatoes (No. 2 can)	739.6	633.4	666.4	622.5		
Lard (lb.)	1387.7	1446.5	1675.8	1722.8		
Sugar (lb.)	1871.5	1708.0	1762.5	1599.0		
Total	58993.0	62233.9	67163.4	72849.5		
Index	100.0	100.0	113.8	117.1		

PROBLEM 64. CONSTRUCTING AN INDEX OF PHYSICAL VOLUME

The Federal Reserve Board of Governors constructs an index series of mineral production in the United States, using the seven minerals listed in Table 44 (plus gold and silver production). The values of this index in recent years are 1946, 134; 1947, 149; 1948, 155; with the period 1935–1939 as the base.

In this problem you will construct an index of mineral production omitting gold and silver, under the assumption that the production of these two products is determined more by monetary, fiscal, and legislative policy than economic activity within the economy.

Prices are given in Table 44 which may be used for weighting purposes and the volumes of production are shown in Table 45.

TABLE 44

PRICE PER UNIT OF MINERAL PRODUCTS
UNITED STATES, AVERAGE PRICES FOR 1947

Anthracite coal	$14.108	per short ton (chestnut, wholesale)
Bituminous coal	6.873	per short ton (mine run, wholesale)
Petroleum	1.843	per bbl. (at well)
Iron ore	5.550	per long ton, non-Bessemer
Copper	0.2096	per lb. (smelter)
Lead	0.1467	per lb. (New York City)
Zinc	0.1050	per lb. (St. Louis, Mo.)

Source: All prices from the *Survey of Current Business*, Annual Supplement, 1949, U. S. Department of Commerce, excepting the iron ore price which came from the Statistical Abstract of the U. S., 1948, p. 300.

PROBLEM 64. (Continued)

TABLE 45

MONTHLY AVERAGE PRODUCTION OF MINERALS—UNITED STATES, 1946–1948

Year	*Anthracite Coal (Thousands of Short Tons)**	*Bituminous Coal (Thousands of Short Tons)**	*Petroleum (Thousands of Barrels)*	*Iron Ore (Thousands of Long Tons Shipped)†*	*Copper, Refined (Short Tons)**	*Zinc-Slab (Short Tons)**	*Lead Refined (Short Tons)**
1946	5,042	44,494	144,495	5,841	50,339	63,279	34,492
1947	4,766	52,552	154,749	7,776	99,699	70,669	48,643
1948	4,754	49,500	168,024	8,444	102,808	70,842	45,439

* A short ton contains 2,000 pounds.
† A long ton contains 2,240 pounds.

Source: *Survey of Current Business*, Annual Supplement, 1949, U. S. Department of Commerce.

Instructions

1. Examine the data carefully and edit them in any way which seems necessary before you begin the computations.

2. Construct an index of physical volume for the three years 1946, 1947, and 1948, using 1947 as the base.

Questions

1. What editing did you consider necessary? Explain how it was done.

2. Are you satisfied that the data given in Tables 44 and 45 are comparable? If not, why not? If not, what biases would the lack of comparable data tend to introduce?

3. What type of index did you construct—a simple aggregative, a simple average of relatives, a weighted aggregative, or a weighted average of relatives? Give the reasons for your choice.

4. Are the movements in your index series similar to those of the Federal Reserve Board index mentioned in the first paragraph of this problem? Would the formula you have used account for any observed differences or do you think the exclusion of gold and silver and minor differences in the quantities employed in your index would account for most of the difference?
See the text for the calculation of the Federal Reserve index.

5. Explain precisely what an increase in your index indicates.

PROBLEM 65. REVIEW OF INDEX NUMBERS

Our problem is to discover what has happened to the composite price of several fats and note how the result differs with the type of index number computed.

The data are shown in Table 46.

PROBLEM 65. (Continued)

TABLE 46

WHOLESALE PRICES OF COTTONSEED OIL, LARD, AND BUTTER, FOR THE MONTH OF OCTOBER, 1946–1949, PRINCIPAL MARKETS, UNITED STATES, AND AVERAGE MONTHLY PRODUCTION, 1947

Commodity	*Average Monthly Production, 1947 Millions of Pounds*	*October Wholesale Price per Pound in Cents*			
		1946	*1947*	*1948*	*1949*
Butter	110.8	84.0	71.8	64.4	62.5
Cottonseed oil	85.8	26.8	23.7	21.5	12.9
Lard	143.5	19.0	28.5	23.4	15.8

Source: Prices and production from *Survey of Current Business*, Annual Supplement, 1949; *ibid.*, February 1950, for 1949 values.

Instructions

1. Construct four index numbers on the 1947 base—a simple aggregative, a simple arithmetic average of relatives, a weighted aggregative and a weighted average of relatives. Organize your computations sytematically and number your tables, Tables 5, 6, 7, 8.

Questions

1. Explain any differences among the results obtained. In making this explanation give a detailed analysis showing the influence of the type of average used and effect of the implicit or explicit weights.
2. Would the results obtained for the weighted aggregative and the weighted average of relatives have been different if total annual production had been used for weighting rather than the average monthly production which was actually employed? Explain.
3. Do you find anything to criticize in the choice of the base year?
4. Which of the four methods of index number construction do you consider best in this case? Explain.

PROBLEM 66. WEEKLY EARNINGS IN MANUFACTURING INDUSTRIES

TABLE 47

ANNUAL AVERAGE OF WEEKLY EARNINGS IN MANUFACTURING INDUSTRIES AND INDEX OF THE COST OF LIVING, UNITED STATES, 1932–1949

Year	*Average Weekly Earnings*	*Consumers' Price Index 1935–39 = 100*	*Weekly Earnings Adjusted for Consumers' Prices*
1932	$17.86	97.6	
1933	17.36	92.4	
1934	18.93	95.7	
1935	20.85	98.1	
1936	22.60	99.1	
1937	24.95	102.7	
1938	22.70	100.8	
1939	23.86	99.4	
1940	25.20	100.2	
1941	29.58	105.2	
1942	36.65	116.5	
1943	43.14	123.6	
1944	46.08	125.5	
1945	44.39	128.4	
1946	43.74	139.3	
1947	49.97	159.2	
1948	54.14	171.2	
1949	54.90_p	169.1	

p = preliminary
Source: Bureau of Labor Statistics, U. S. Department of Labor.

Instructions

1. Compute the change in the average weekly real income of American industrial workers for the eighteen-year period.

Questions

1. Would a deflated index series of average hourly earnings move identically with this series of average weekly earnings? Why?
2. As a statistician employed by Ford at the Kansas City plant, could you use the Consumers' Price index given in this table in adjusting weekly or hourly earnings in the plant, to discover whether the real income of the workers had gone up or down? Explain.
3. Write a paragraph explaining the meaning of the term "real" wages as derived in the above calculations.
4. The sales of the Acme Corporation declined from $470,000 in 1948 to $376,000 in 1949. During the same period, an index number of the prices of things sold by the Acme Corporation declined from 100 (1948) to 80 (1949). It is clear that the physical volume of merchandise sold was considerably less in 1949 than in 1948. Do you agree with the conclusion?

SECTION IX

Time Series Analysis

1. The Measurement of Trend (Chapter XIII of text), Problems 67, 68, 69, 70, 71
2. The Analysis of Seasonal Variations (Chapter XIV of text), Problems 72, 73, 74
3. Adjustment for Trend (Chapter XV of text), Problems 75, 76, 77
4. Adjustment for Trend and Seasonal Variation (Chapter XV of text), Problems 78, 79, 80
5. Summary and Review, Problem 81

Measurement of Trend

PROBLEM 67. CALCULATION OF TREND AS A FIRST STEP IN ISOLATION OF THE CYCLE

In Table 48 are shown the data for production of Portland Cement in the United States, 1890–1949. These data are also shown in Chart 2.

A trend expression for the entire statistical history of this industry would be of the type discussed in the text under the heading "Growth Curves" (pp. 535–538). Assume, however, that we now merely wish to examine the cyclical swings in this series and for this purpose a straight line fitted to the data of recent years will serve as a base from which to measure the cyclical variations.

Instructions

1. Select the period to be included in the analysis of trend so that the purposes stated in the preceding paragraph can be accomplished.
2. Fit an arithmetic straight line, method of least squares, to the data for the period you have chosen and plot the line on Chart 2.
3. Turn to Problem 75 and record your trend values in Table 58, column *b*. In Problem 75 the cycle will be isolated.

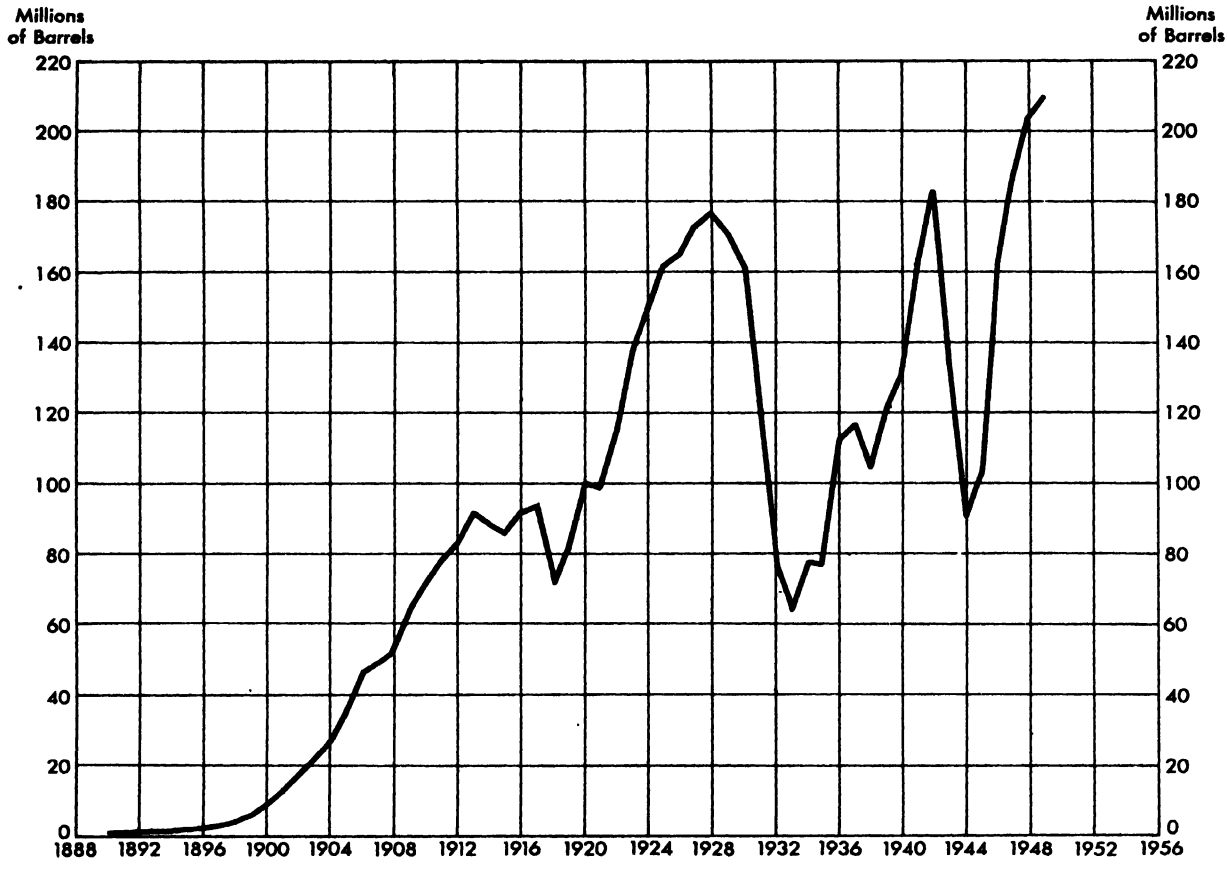
Questions

1. Criticize the use of a straight line trend to describe the growth characteristics of this industry during its entire statistical history, 1890 to date.
2. What factors influenced you in your choice of a period for straight line trend analysis? Explain.
3. Explain the meaning of the *b*-value in the trend equation.
4. If your objective had been to compute a trend for purposes of extrapolation, would you have used the arithmetic straight line? Explain.

PROBLEM 67. (Continued)

CHART 2

ANNUAL PRODUCTION OF PORTLAND CEMENT
UNITED STATES, 1890–1949



Source: Table 48.

PROBLEM 67. (Continued)

TABLE 48

ANNUAL PRODUCTION OF PORTLAND CEMENT, UNITED STATES
1890–1949

Year	*Portland Cement (000 bbls.)*	*X*	X^2	*XY*	*Trend*
1890	335				
1891	455				
1892	547				
1893	591				
1894	799				
1895	999				
1896	1,543				
1897	2,678				
1898	3,692				
1899	5,652				
1900	8,482				
1901	12,711				
1902	17,231				
1903	22,343				
1904	26,506				
1905	35,247				
1906	46,463				
1907	48,785				
1908	51,073				
1909	64,991				
1910	76,550				
1911	78,529				
1912	82,438				
1913	92,097				
1914	88,230				
1915	85,915				
1916	91,521				
1917	92,814				
1918	71,082				
1919	80,778				
1920	100,023				
1921	98,842				
1922	114,790				
1923	137,460				
1924	149,358				

PROBLEM 67. (Continued)

TABLE 48 (Continued)

ANNUAL PRODUCTION OF PORTLAND CEMENT, UNITED STATES
1890–1949

Year	*Portland Cement (000 bbls.)*	*X*	X^2	*XY*	*Trend*
1925	161,659				
1926	164,530				
1927	173,207				
1928	176,299				
1929	170,646				
1930	160,908				
1931	124,572				
1932	76,512				
1933	63,372				
1934	77,688				
1935	76,476				
1936	112,368				
1937	116,484				
1938	105,552				
1939	121,824				
1940	130,296				
1941	164,004				
1942	182,760				
1943	133,488				
1944	90,840				
1945	102,816				
1946	163,800				
1947	186,528				
1948	205,428				
1949	209,831				
Totals					

Source: Data for the period 1890–1934, *Statistical Abstract of the United States, 1940;* p. 798: for 1935–1948, *Survey of Current Business*, Statistical Supplement, 1949, p. 182, (Monthly average×12): for 1949, *Survey of Current Business*, February, 1950, p. S-18.

PROBLEM 68. TREND OF TRUCK PRODUCTION, MONTHLY AVERAGE, U. S. FACTORY SALES

Over the period 1913 through 1941, the straight-line trend of monthly average truck production, in thousands of trucks, as calculated by the least squares method was $Y_c = 36.66 + 2.28X$; X equals one year, origin 1927.

PROBLEM 68. (Continued)

Instructions

Plot the data on U. S. truck production 1913–1941 as contained in Table 49 and draw in the 1913–1941 trend described above. Label your chart, Chart 19.

Questions

1. Does the least squares prewar trend statement provide an adequate description of the trend of monthly average truck production for the postwar period? Why or why not?

2. What are the procedures you would follow in preparing a forecast of truck production for 1952? As a part of your answer indicate the assumptions you would make with respect to the forecast year.

TABLE 49

U. S. TRUCK PRODUCTION, FACTORY SALES, MONTHLY AVERAGE
1913–1949

Year	*Thousands of Trucks*	*Year*	*Thousands of Trucks*
1913	2.0	1931	34.7
1914	2.1	1932	19.6
1915	6.2	1933	28.9
1916	7.7	1934	47.9
1917	10.7	1935	57.9
1918	18.9	1936	65.4
1919	23.0	1937	74.4
		1938	40.7
1920	26.8	1939	59.2
1921	12.3		
1922	22.5	1940	62.4
1923	34.1	1941	87.8
1924	34.7	1942	15.4
1925	44.2	1943	2.1
1926	43.1	1944	9.7
1927	38.7	1945	24.3
1928	45.3	1946	77.6
1929	64.3	1947	101.7
		1948	113.7
1930	47.6	1949	93.6

Source: *Survey of Current Business*, Statistical Supplements for 1942, 1947, and 1949, and March 1950 issue.

PROBLEM 69. TREND ANALYSIS APPLIED TO ANNUAL PRODUCTION OF PIG IRON

Pig iron production is one of the basic time series in our industrial economy, and the pattern of its cyclical activity has been quite similar to the fluctuations of indexes purporting to show the condition of general business. In this problem, we propose to measure the growth of pig iron production since 1919 by means of trend analysis. The production of pig iron in the United States is shown in long tons for the period 1919 to 1949 in Table 50.

Instructions

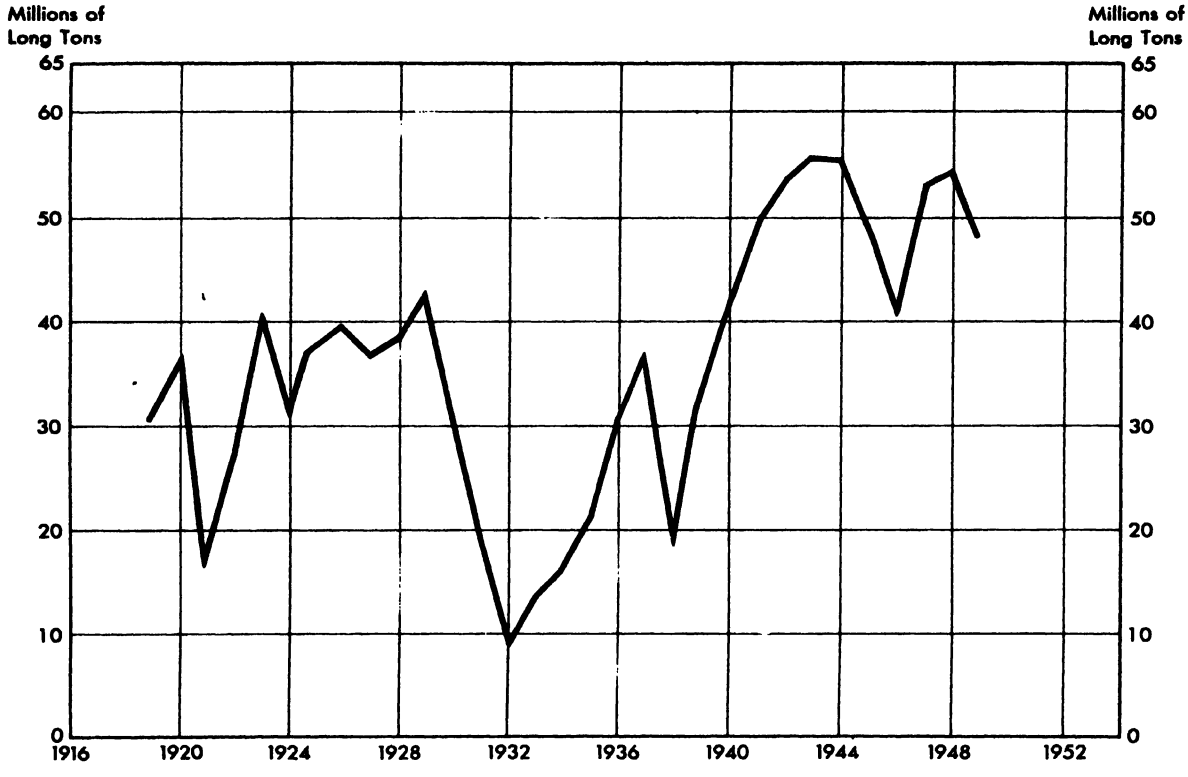
1. Calculate an arithmetic straight line trend using the method of least squares, selecting the period to be included in the analysis which will give a satisfactory trend for pig iron production in the United States. Show your calculations and write the equation for such trend at the bottom of Table 50.

2. Plot the trend you calculated on Chart 3.

PROBLEM 69. (Continued)

CHART 3

ANNUAL PRODUCTION OF PIG IRON
UNITED STATES, 1919–1949



Source: Table 50.

PROBLEM 69. (Continued)

TABLE 50

ANNUAL PRODUCTION OF PIG IRON IN THE UNITED STATES
1919–1949

Year	*Pig Iron (000 long tons)*	*X*	X^2	*XY*	*Trend*
1919	30,588				
1920	36,420				
1921	16,548				
1922	26,880				
1923	40,056				
1924	31,104				
1925	36,408				
1926	39,072				
1927	36,228				
1928	37,836				
1929	42,288				
1930	31,404				
1931	18,276				
1932	8,688				
1933	13,212				
1934	15,912				
1935	21,010				
1936	30,621				
1937	36,611				
1938	18,782				
1939	31,532				
1940	41,914				
1941	49,918				
1942	53,561				
1943	55,157				
1944	55,307				
1945	48,364				
1946	40,521				
1947	52,864				
1948	54,332				
1949	48,399				
Totals					

Source: Prior to 1942, these data were compiled by *The Iron Age*, subsequently by *American Iron and Steel Institute*. Data are substantially comparable. Taken from, *Survey of Current Business*, 1940 Supplement, p, 130 for the period 1919–1934 (Monthly averages×12); *ibid.* Statistical Supplement, 1949, p. 157 for the period 1935–1948 (converted from short tons); *ibid.* February, 1950, p. S-32 for 1949.

Computed Trend: Y_c=_____+_____X: Origin _____, X=_____.

PROBLEM 69. (Continued)

3. Project the trend through 1952.

4. Calculate by the semi-average method a trend for the same period you used in Instruction 1, and plot it on Chart 3.

5. Turn to Problem 76 and record the trend computed in Instruction 1 in Table 59, column *b*.

Questions

1. What cautions did you observe in selecting the period for trend analysis in pig iron production?

2. Explain the meaning of the *b*-value in your trend equation and any limitations in its use for long-term projection.

3. Are your least squares trend and your semi-average trend identical? If not, why do they differ and which do you recommend as the better suited for analytical purposes?

4. Write a paragraph explaining what use this trend might be in evaluating the adequacy of pig iron production in an expanding economy.

PROBLEM 70. STRAIGHT-LINE ARITHMETIC TREND, ANTHRACITE COAL PRODUCTION

Instructions

1. Refer to Table 51 and calculate an arithmetic straight-line trend, method of least squares. Select the period to be included in the analysis which you think will give the best expression of trend for anthracite coal production in Pennsylvania (see Chart 4). Show all calculations in systematic form and write the equation for your trend line on the bottom of Table 51.

2. Plot the trend you have calculated on Chart 4.

3. Calculate a trend for the same period used in Question 1 by the semi-average method and plot it on Chart 4. Label the curves so that they may be easily distinguished.

4. Turn to Problem 77 and record the trend in Table 60, column *b*.

Questions

1. Did you fit an arithmetic trend to the entire period show in Table 51? If not, why not and on what basis did you determine the period to be fitted?

2. How would you explain the meaning of the *b*-value to an association or group interested in the future of anthracite?

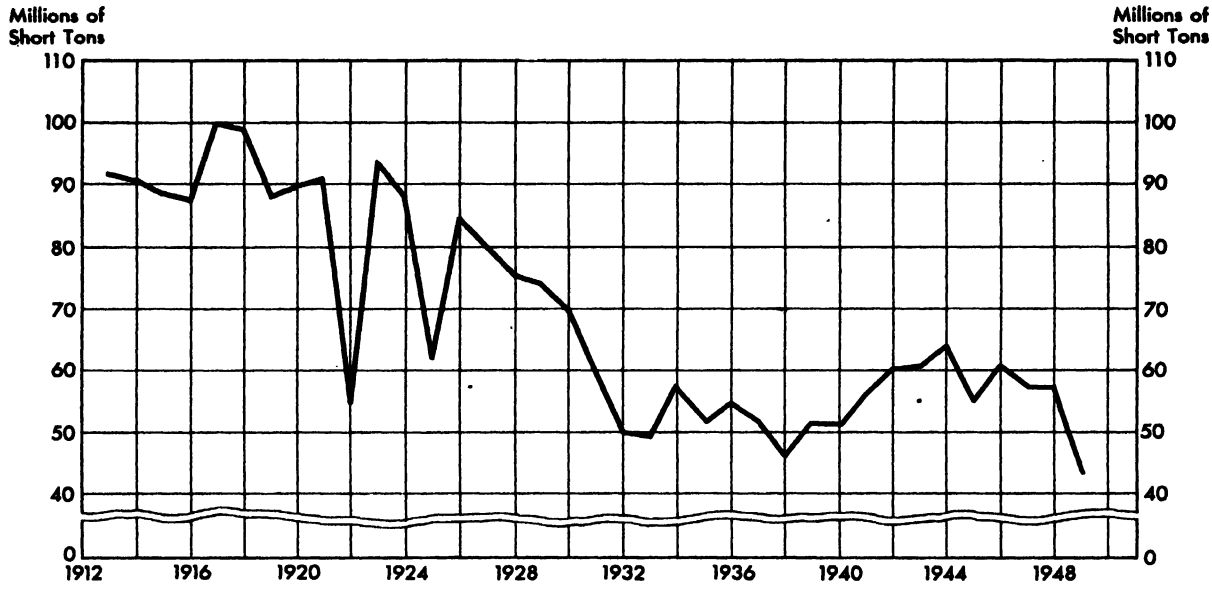
3. Which trend would you defend as the better for extrapolation purposes, the least squares or the semi-average? Why?

4. Contrast the limitations to be considered in the use of trend for short-time forecasts and for long-time forecasts.

PROBLEM 70. (Continued)

CHART 4

ANNUAL ANTHRACITE COAL PRODUCTION
PENNSYLVANIA, 1913–1949



Source: *Statistical Abstract of the United States*, 1940, p. 784 for years 1913–1934; *Survey of Current Business*, Statistical Supplement for 1949, p. 168 for the years 1935 through 1948. *Ibid.*, February 1950, p. S-34 for 1949 data.

PROBLEM 70. (Continued)

TABLE 51

ANNUAL ANTHRACITE COAL PRODUCTION IN PENNSYLVANIA
1916–1948

Year	*Anthracite Production (000 short tons)*	*X*	X^2	*XY*	*Trend*
1916	87,578				
1917	99,612				
1918	98,826				
1919	88,092				
1920	89,598				
1921	90,473				
1922	54,683				
1923	93,339				
1924	87,927				
1925	61,817				
1926	84,437				
1927	80,096				
1928	75,348				
1929	73,828				
1930	69,385				
1931	59,646				
1932	49,855				
1933	49,541				
1934	57,168				
1935	52,164				
1936	54,576				
1937	51,852				
1938	46,104				
1939	51,492				
1940	51,480				
1941	56,364				
1942	60,324				
1943	60,648				
1944	63,696				
1945	54,936				
1946	60,504				
1947	57,192				
1948	57,048				

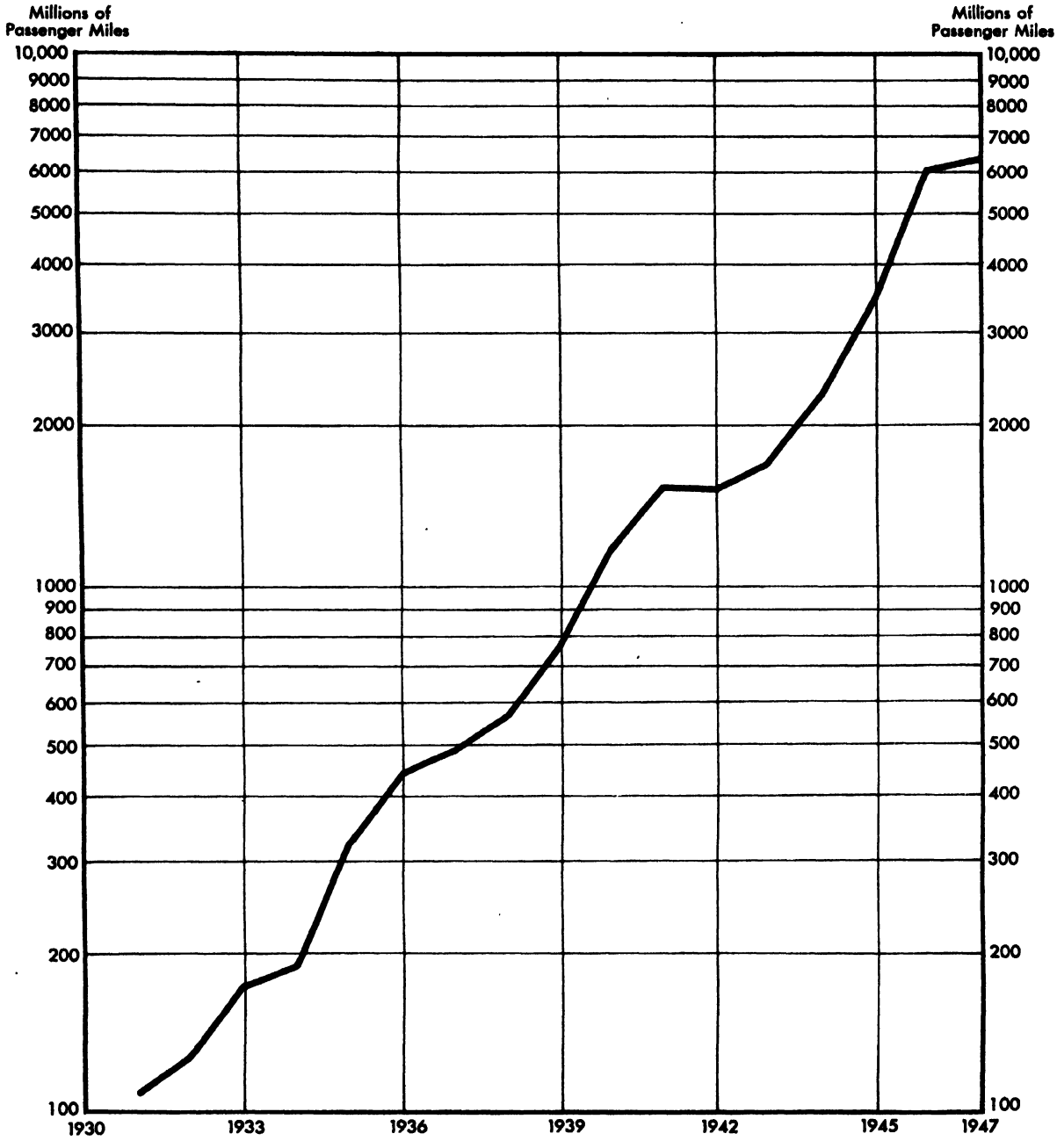
Source: *Statistical Abstract of the United States*, 1940, p. 784 for years 1916–1934; *Survey of Current Business* Statistical Supplement for 1949, p. 168 for the years 1935 through 1948.

Estimated Trend: $Y_c =$ _____ + _____ X. Origin _____ $X =$ _____.

PROBLEM 70. (Continued)

CHART 5

PASSENGER MILES FLOWN BY SCHEDULED AIRLINES
UNITED STATES, 1931–1947



Source: Table 52.

PROBLEM 71. STRAIGHT-LINE TREND

The data for passenger miles flown by scheduled airlines in the United States are shown in the table below. Of particular interest is the steady rate of growth throughout the depression and the war period.

TABLE 52

PASSENGER MILES FLOWN BY SCHEDULED AIRLINES IN THE UNITED STATES
1931–1947

Year	*Monthly Averages (millions of miles)*	*Log Y*	*X*	X^2	*X log Y*	*Log of Trend*	*Trend*
1931	106.9						
1932	127.4						
1933	174.8						
1934	189.8						
1935	316.3						
1936	439.0						
1937	481.1						
1938	560.7						
1939	755.1						
1940	1,157.9						
1941	1,506.3						
1942	1,501.3						
1943	1,670.9						
1944	2,211.9						
1945	3,408.3						
1946	6,068.3						
1947	6,307.7						
1948							
1949							
1950							

Source: *Statistical Handbook of Civil Aviation*, U. S. Department of Commerce, 1948, p. 67, for 1931–1947.

Instructions

1. Analyze the data in Table 52 and as they appear in Chart 5. Decide what type of trend seems best for straight line analysis.
2. Fit a straight-line trend to these data.
3. Ascertain the percentage increase per year in this series for the period included in your analysis.
4. Plot the trend you have computed on Chart 5.

Questions

1. What is the main difference between an arithmetic straight line of trend and a geometric straight line?

PROBLEM 71. (Continued)

2. Would this trend (geometric) show negative values if extended back to early years? Would an arithmetic line show negative values if pushed back through time?

3. How can you tell whether to fit an arithmetic or geometric trend to a series of data?

4. Do you think this industry can expect to maintain, in the future, the rate of expansion these data show? Explain.

Analysis of Seasonal Variations

PROBLEM 72. RATIO TO MOVING AVERAGE METHOD OF SEASONAL ANALYSIS APPLIED TO FROZEN FRUITS, STOCKS,COLD STORAGE, END OF THE MONTH

During the period covered by these data, the market for frozen fruits underwent marked changes. In the early years of the war this product was rationed. Soon, however, cold storage space became congested and frozen fruits were de-rationed in 1943 in the hope that old stocks could be moved rapidly to make storage space for new crops. The quantity of different kinds of fruit harvested each year is subject to marked variation but not all fruits are equally affected by differences in growing conditions and there appears to be considerable stability in the total processing.

Unlike other production series which show no seasonal movement during the war years when output was held at capacity for prolonged periods, this series reveals a reasonably stable pattern throughout in spite of the changing conditions in the economy and in the marketing of this commodity. See Chart 6.

Instructions

1. Turn to Table 53 and complete the calculations which remain to be done.
2. Plot your extension of the centered moving average in Chart 6.
3. Complete the computations in Table 54 for May and November as indicated.
4. Level the crude seasonal index. These final percentages are the seasonal index.
5. Examine Chart 7 month by month and judge whether the dispersion of the specific seasonal percentages above and below the typical seasonal percentage is such as to limit its usefulness?

Questions

1. Examine the centered moving average which you have plotted on Chart 6. Has the influence of month-to-month variation within a twelve-month period been greatly reduced in the calculation of this line? Why?

2. Of the four components of time series, seasonal, cycle, trend, and irregular factors, which have been largely taken from the ratios in column *e*, Table 53?

3. Why is the positional mean taken of the ratios of specific seasonal percentages rather than an arithmetic mean of all items?

4. If you were an executive in a company marketing frozen fruits, what practical use would you find for an adequate analysis of seasonal fluctuations? Be specific in your answers.

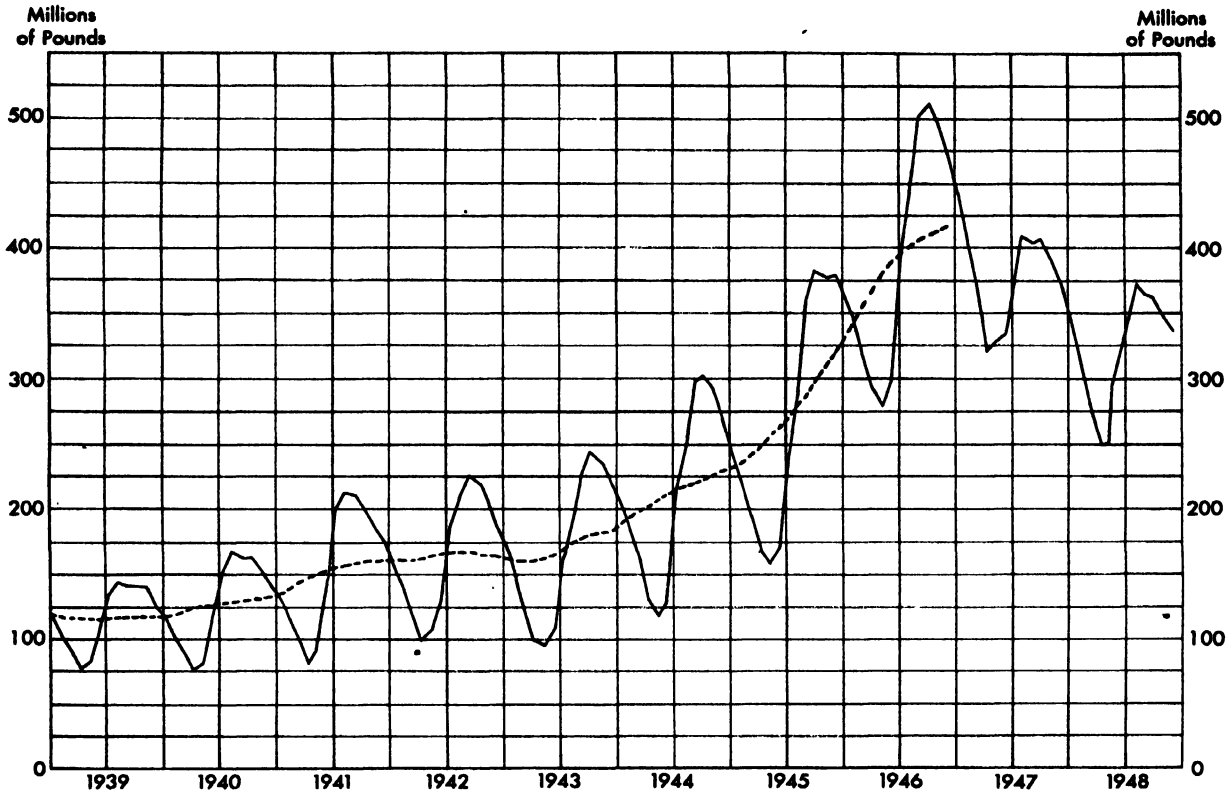
5. What do you consider to be the limitations of the index "of typical seasonal variation" which you have computed?

6. What tests should be applied before any index of seasonal variation is accepted as satisfactory?

PROBLEM 72. (Continued)

CHART 6

FROZEN FRUIT STOCKS IN COLD STORAGE END OF MONTH, UNITED STATES 1938–1949 AND CENTERED 12-MONTH MOVING AVERAGE



Source: Table 53.

PROBLEM 72. (Continued)

TABLE 53

SEASONAL ANALYSIS APPLIED TO FROZEN FRUITS, STOCKS, COLD STORAGE, END OF THE MONTH

	1938					*1939*				
	Monthly Stocks	*12-Month Total*	*12-Month Average*	*Centered 12-Month Average*	*Ratio a÷d*	*Monthly Stocks*	*12-Month Total*	*12-Month Average*	*Centered 12-Month Average*	*Ratio a÷d*
	a	*b*	*c*	*d*	*e*	*a*	*b*	*c*	*d*	*e*
Jan.						116.8	1,414.0	117.8	118.7	98.4
Feb.						103.1	1,405.7	117.1	117.4	87.8
March						91.1	1,400.2	116.7	116.9	77.9
April						76.2	1,398.8	116.6	116.6	65.4
May						83.7	1,402.1	116.8	116.7	71.7
June						104.1	1,404.6	117.0	116.9	89.0
July	152.9					132.1	1,405.1	117.1	117.1	112.8
Aug.	151.4					143.1	1,406.9	117.2	117.2	122.1
Sept.	147.6					142.1	1,405.5	117.1	117.2	121.2
Oct.	143.8					142.4	1,405.4	117.1	117.1	121.6
Nov.	138.3					141.6	1,404.1	117.0	117.0	121.0
Dec.	125.8					128.3			117.7	109.0
		1,434.8	119.6				1,420.9	118.4		

TABLE 53 (Continued)

SEASONAL ANALYSIS APPLIED TO FROZEN FRUITS, STOCK, COLD STORAGE, END OF THE MONTH

	1940					*1941*				
	Monthly Stocks	*12-Month Total*	*12-Month Average*	*Centered 12-Month Average*	*Ratio a÷d*	*Monthly Stocks*	*12-Month Total*	*12-Month Average*	*Centered 12-Month Average*	*Ratio a÷d*
	a	*b*	*c*	*d*	*e*	*a*	*b*	*c*	*d*	*e*
Jan.	117.3			119.2	98.4	128.3			135.7	94.5
Feb.	104.9	1,439.8	120.0	121.0	87.6	114.9	1,562.1	137.8	139.6	82.3
March	89.7	1,463.1	121.9	122.8	73.0	99.4	1,698.0	141.5	143.6	69.2
April	76.1	1,483.8	123.6	124.5	61.1	81.9	1,746.7	145.6	147.1	55.7
May	82.4	1,505.2	125.4	126.0	65.4	91.4	1,782.7	148.6	151.0	60.5
June	120.9	1,518.3	126.5	127.1	95.1	145.7	1,814.7	153.5	153.8	94.5
July	151.0	1,532.5	127.7	128.1	117.9	200.3	1,850.7	154.2	155.4	128.9
Aug.	166.4	1,543.5	128.6	129.0	129.0	212.3	1,879.8	156.6	157.8	134.5
Sept.	162.8	1,553.5	129.5	129.9	125.3	211.5	1,907.1	158.9	159.8	132.4
Oct.	163.8	1,563.2	130.3	130.5	125.5	199.8	1,927.7	160.6	161.4	123.8
Nov.	154.7	1,569.0	130.7	141.1	118.0	186.7	1,946.8	162.2	162.8	114.7
Dec.	142.5	1,578.0	131.5	132.6	107.5	177.9	1,961.9	163.5	162.3	109.6
		1,602.8	133.6				1,945.5	162.1		

PROBLEM 72. (Continued)

TABLE 53 (Continued)

SEASONAL ANALYSIS APPLIED TO FROZEN FRUITS, STOCKS, COLD STORAGE, END OF THE MONTH

	1942					*1943*				
	Monthly Stocks	*12-Month Total*	*12-Month Average*	*Centered 12-Month Average*	*Ratio a÷d*	*Monthly Stocks*	*12-Month Total*	*12-Month Average*	*Centered 12-Month Average*	*Ratio a÷d*
	a	*b*	*c*	*d*	*e*	*a*	*b*	*c*	*d*	*e*
Jan.	158.0			161.5	97.8	172.1			163.9	105.0
		1,931.2	160.9				1,955.4	162.9		
Feb.	142.2			161.8	87.9	145.3			162.0	89.7
		1,926.7	160.6				1,932.4	161.0		
March	120.0			161.2	74.4	124.4			161.0	77.3
		1,940.3	161.7				1,931.3	160.9		
April	101.0			162.6	62.1	99.0			161.8	62.1
		1,962.2	163.5				1,953.1	162.8		
May	106.0			164.4	64.9	96.5			164.1	58.8
		1,981.9	165.2				1,985.0	165.4		
June	129.3			165.0	78.1	107.1			167.0	64.1
		1,992.0	166.0				2,024.0	168.7		
July	186.0			166.6	111.6	162.0			170.2	95.2
		2,006.1	167.2				2,061.7	171.8		
Aug.	207.8			167.3	124.2	184.8			173.5	106.5
		2,009.2	167.4				2,102.5	175.2		
Sept.	225.1			167.6	134.3	224.0			176.8	126.7
		2,013.6	167.8				2,139.7	178.3		
Oct.	221.7			167.7	132.2	243.5			179.6	135.6
		2,011.6	167.6				2,171.6	181.0		
Nov.	206.4			167.2	122.7	238.3			181.8	131.1
		2,001.6	166.8				2,192.0	182.7		
Dec.	188.0			165.8	113.4	227.0			183.6	123.6
		1,979.4	164.9				2,214.4	184.5		

TABLE 53 (Continued)

SEASONAL ANALYSIS APPLIED TO FROZEN FRUITS, STOCKS, COLD STORAGE, END OF THE MONTH

	1944					*1945*				
	Monthly Stocks	*12-Month Total*	*12-Month Average*	*Centered 12-Month Average*	*Ratio a÷d*	*Monthly Stocks*	*12-Month Total*	*12-Month Average*	*Centered 12-Month Average*	*Ratio a÷d*
	a	*b*	*c*	*d*	*e*	*a*	*b*	*c*	*d*	*e*
Jan.	209.8			186.7	112.4	242.3			232.0	104.4
		2,266.9	188.9				2,796.6	233.0		
Feb.	186.1			191.4	97.2	217.0			234.8	92.4
		2,328.6	194.0				2,838.9	236.6		
March	161.6			197.1	82.0	193.8			243.6	79.6
		2,402.7	200.2				2,901.0	241.7		
April	130.9			202.6	64.6	168.9			245.0	68.9
		2,460.8	205.1				2,980.7	248.4		
May	116.0			207.3	56.4	159.4			252.0	63.2
		2,513.7	209.5				3,066.6	255.5		
June	129.5			211.2	61.3	169.5			260.1	65.2
		2,555.2	212.9				3,176.9	264.7		
July	214.5			214.2	100.1	239.8			269.7	88.9
		2,587.7	215.6				3,296.9	274.7		
Aug.	246.5			216.9	113.6	288.8			280.0	103.1
		2,618.6	218.2				3,423.9	285.3		
Sept.	298.1			219.6	135.7	360.2			290.6	124.0
		2,650.8	220.9				3,551.9	296.0		
Oct.	301.6			222.5	135.6	381.3			301.1	126.6
		2,688.8	224.1				3,674.1	306.2		
Nov.	291.2			225.8	120.0	377.1			311.2	121.2
		2,731.3	227.6				3,792.8	316.1		
Dec.	268.5			229.2	117.1	378.8			321.4	117.8
		2,771.3	230.9				3,920.9	326.7		

PROBLEM 72. (Continued)

TABLE 53 (Continued)

SEASONAL ANALYSIS APPLIED TO FROZEN FRUITS, STOCKS, COLD STORAGE, END OF THE MONTH

	1946					*1947*				
	Monthly Stocks	*12-Month Total*	*12-Month Average*	*Centered 12-Month Average*	*Ratio a÷d*	*Monthly Stocks*	*12-Month Total*	*12-Month Average*	*Centered 12-Month Average*	*Ratio a÷d*
	a	*b*	*c*	*d*	*e*	*a*	*b*	*d*	*d*	*e*
Jan.	362.3					439.2				
Feb.	344.0					403.7				
March	321.8					367.0				
April	291.1					319.7				
May	278.1					327.7				
June	297.6					331.3				
July	396.6					374.4				
Aug.	459.6					408.1				
Sept.	501.9					402.8				
Oct.	510.3					405.8				
Nov.	497.8					392.0				
Dec.	470.7					369.5				

TABLE 53 (Continued)

SEASONAL ANALYSIS APPLIED TO FROZEN FRUITS, STOCKS, COLD STORAGE, END OF THE MONTH

	1948					*1949*				
	Monthly Stocks	*12-Month Total*	*12-Month Average*	*Centered 12-Month Average*	*Ratio a÷d*	*Monthly Stocks*	*12-Month Total*	*12-Month Average*	*Centered 12-Month Average*	*Ratio a÷d*
	a	*b*	*c*	*d*	*e*	*a*	*b*	*c*	*d*	*e*
Jan.	343.5					317.7				
Feb.	316.8					301.2				
March	281.8					266.6				
April	247.9					237.4				
May	250.3					237.9				
June	280.7					255.8				
July	340.9					327.1				
Aug.	371.6									
Sept.	364.1									
Oct.	362.4									
Nov.	346.9									
Dec.	335.7									

Source: *Survey of Current Business*, Supplement, 1942, for years 1938–1941, p. 117; *ibid.*, Supplement, 1947, for years 1942–1946, p. 127; *ibid.*, Supplement, 1949, for years 1947–1948, p. 128; *ibid.*, February 1950 for 1949, p. S-27.

PROBLEM 72. (Continued)

TABLE 54

ARRAY OF MONTHLY RATIOS OF FROZEN FRUITS, STOCKS, COLD STORAGE, END OF MONTH TO MOVING AVERAGES
UNITED STATES, 1939–1948

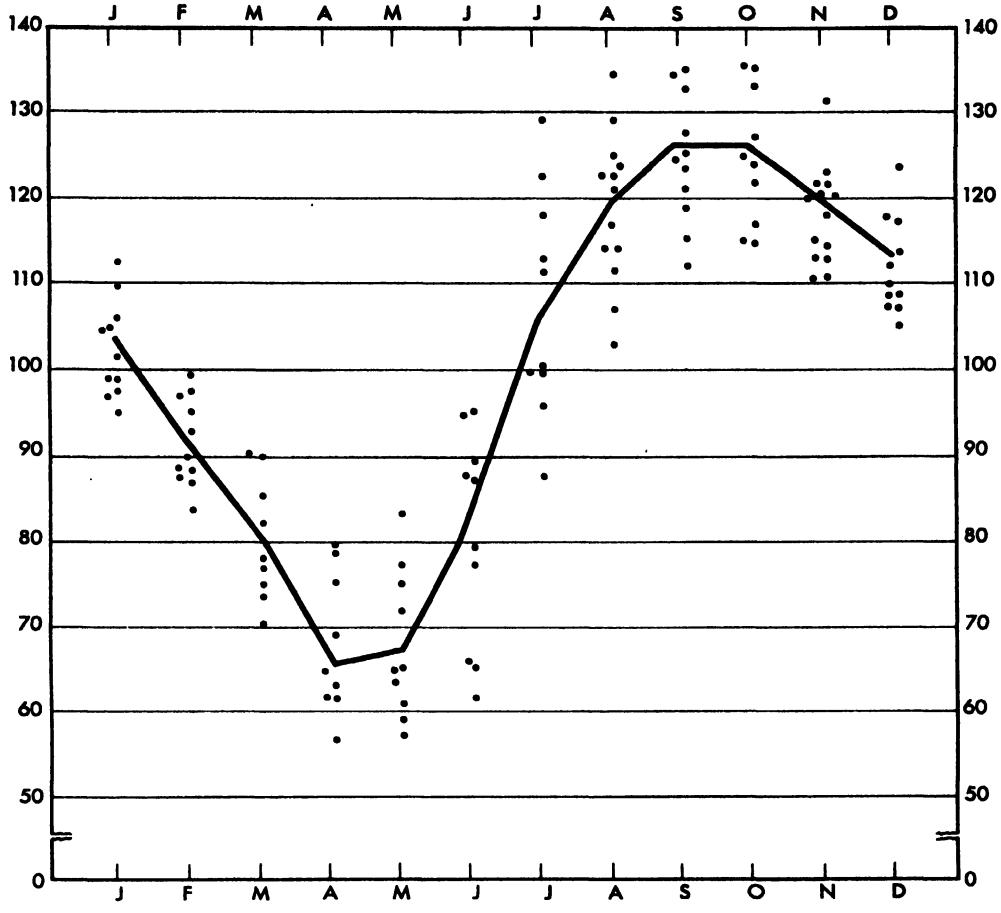
	Jan.	*Feb.*	*March*	*April*	*May*	*June*	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*
	112.4	99.2	89.8	79.9		95.1	128.9	124.5	135.7	135.6		123.6
	109.4	97.3	89.4	78.4		94.5	117.9	129.0	134.3	135.6		117.8
	105.1	97.2	84.9	75.4		89.0	112.8	124.2	132.4	132.2		117.1
	105.0	94.5	82.0	68.9		87.3	111.6	122.1	126.7	126.6		113.4
	104.4	92.4	79.6	65.4		86.8	106.8	117.1	125.3	125.5		112.7
	101.6	89.7	77.9	64.6		78.1	100.1	114.0	124.1	124.3		109.6
	98.4	87.9	77.3	62.1		76.2	100.0	113.6	123.1	123.8		109.0
	98.4	87.8	74.4	61.2		65.2	99.8	111.2	121.2	121.6		108.2
	97.8	86.7	73.0	61.1		64.1	95.2	106.5	115.1	115.0		107.5
	94.5	82.3	69.2	55.7		61.3	88.9	103.1	112.0	114.9		107.4
Total	409.4	364.5	316.8	261.0		328.4	418.5	466.8	499.2	500.2		444.7
Mean												
Final seasonal index												

Source: Table 53.

PROBLEM 72. (Continued)

CHART 7

SPECIFIC SEASONAL RATIOS AND THE SEASONAL INDEX
FROZEN FRUIT STOCKS IN COLD STORAGE
UNITED STATES, 1938–1948



Source: Table 54.

PROBLEM 72. (Continued)

PROBLEM 73. CHANGING PATTERN OF SEASONAL VARIATION

In Problem 72 a typical seasonal index has been computed. The ratios used in the construction of that index are now to be examined to determine whether there has been a pattern in their variations which would cast doubt on the validity of a single index of seasonal variation.

All ratios to the moving average computed in Problem 72 have been assembled in Table 55 and plotted in Chart 8.

Instructions

Analyze the data in both table and chart and answer the following questions.

Questions

1. Do you see any evidence of a changing seasonal pattern? If yes, does it appear to be a gradual change or an abrupt one?
2. Write a paragraph describing the changes, if any, which you see in the seasonal pattern.
3. Is the change, if any, such as to lead you to discard the "typical" seasonal index computed in Problem 72?
4. If the seasonal pattern seems to be changing, what method would you use to obtain satisfactory results?
5. What tests should be applied to any single index of seasonal variation or any set of such indexes before they are finally accepted as satisfactory.

TABLE 55

MONTHLY RATIOS OF FROZEN FRUITS, STOCKS, COLD STORAGE, END OF MONTH TO MOVING AVERAGES
UNITED STATES, 1939–1948

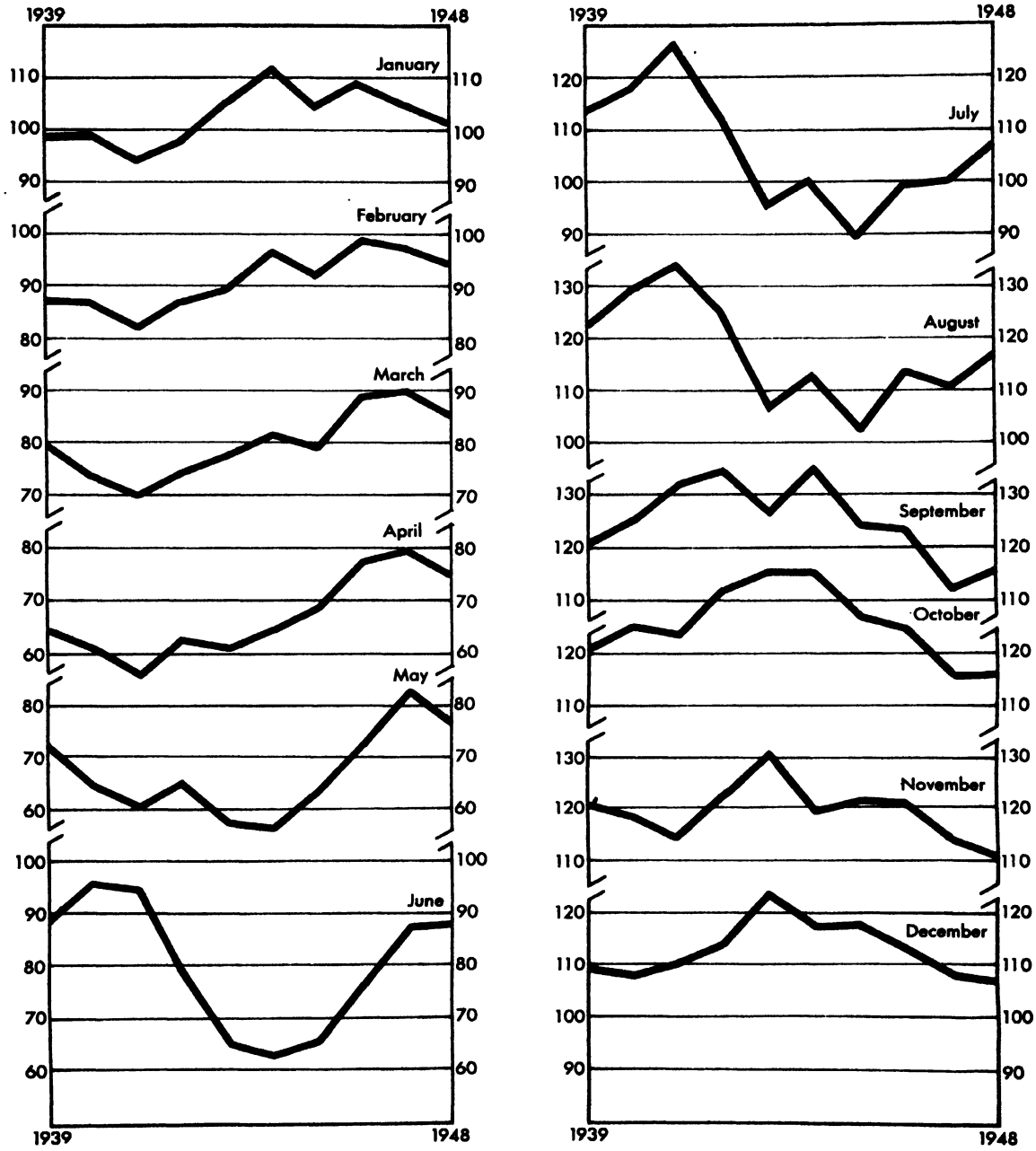
	Jan.	*Feb.*	*March*	*April*	*May*	*June*	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*
1939	98.4	87.8	77.9	65.4	71.0	89.0	112.8	122.1	121.2	121.6	121.0	109.0
1940	98.4	86.7	73.0	61.1	65.4	95.1	117.9	129.0	125.3	125.5	118.0	107.5
1941	94.5	82.3	69.2	55.7	60.5	94.5	128.9	134.5	132.4	123.8	114.7	109.6
1942	97.8	87.9	74.4	62.1	64.9	78.1	111.6	124.2	134.3	132.2	122.7	113.4
1943	105.0	89.7	77.3	61.2	58.8	64.1	95.2	106.5	126.7	135.6	131.1	123.6
1944	112.4	97.2	82.0	64.6	56.4	61.3	100.1	113.5	135.7	135.6	120.0	117.1
1945	104.4	92.4	79.6	68.9	63.2	65.2	88.9	103.1	124.0	126.6	121.2	117.8
1946	109.4	99.2	89.4	78.4	72.9	76.2	99.8	114.0	123.1	124.3	120.3	112.7
1947	105.1	97.3	89.8	79.9	83.7	86.8	100.0	111.2	112.0	114.9	113.0	108.2
1948	101.6	94.5	84.9	75.4	77.0	87.3	106.8	117.2	115.1	115.0	110.4	107.4

Source: Table 53.

PROBLEM 73. (Continued)

CHART 8

THE CHANGING SEASONAL PATTERN FROZEN FRUIT STOCKS IN COLD STORAGE
UNITED STATES, 1939–1948



Source: Table 55.

PROBLEM 74. SEASONAL INDEX OF OIL BURNER SHIPMENTS

Shipments of oil burners in the United States had a definite seasonal pattern in the prewar period. Burner production was limited during the war years and a substantial backlog of demand for oil burners was manifest in 1946, 1947, and 1948. In these years the demand for oil burners at prevailing prices was so much greater than their production that output was expanded and held at high levels through most of this period, and it was not possible to discern a stable seasonal pattern. In 1949, production seemed to have caught up and once more a seasonal pattern in the shipments of oil burners appeared to emerge.

Table 56 contains the data for monthly shipments of oil burners 1934 through 1949. Table 57 contains a seasonal index based on the 1934–1941 period.

Assume that you are employed by a large oil burner manufacturer to develop a seasonal index for oil burner shipments which can be used of the years subsequent to 1949.

Instructions

1. Plot as many values from Table 57 as are necessary to reveal to you the pre-war monthly pattern, the movements during the war period and the post-war monthly movements. Label your chart, Chart 20.

2. Using the prewar seasonal index, deseasonalize the 1948 and 1949 actual shipments and plot on Chart 20.

3. Prepare a tentative seasonal index of oil burner shipments for the period subsequent to 1949. Use your best judgment in determining what procedures to use.

TABLE 56

SHIPMENTS OF OIL BURNERS 1934–1939
Thousands

Year	*Jan.*	*Feb.*	*Mar.*	*Apr.*	*May*	*June*	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*
1934	3.5	3.0	5.1	6.7	8.1	5.9	7.1	12.1	17.0	18.0	8.8	5.5
1935	4.4	4.6	5.7	8.6	9.7	10.0	9.7	15.5	27.1	25.7	10.0	7.7
1936	7.7	7.3	8.9	10.2	12.4	14.7	17.5	21.2	35.4	34.0	16.3	11.3
1937	9.3	9.8	14.4	14.7	14.4	14.7	16.4	22.4	33.7	24.5	10.5	8.2
1938	6.3	5.5	7.9	7.3	8.7	9.6	10.7	15.4	26.4	21.1	11.5	8.8
1939	8.7	7.7	10.7	10.6	15.0	16.9	17.3	24.7	35.4	34.7	20.1	13.3
1940	13.0	11.5	12.8	14.4	17.8	18.4	22.0	31.5	41.5	40.6	24.2	16.5
1941	16.2	16.1	18.2	22.8	28.8	32.7	27.8	31.4	34.7	31.4	21.8	21.9
1942	19.2	18.0	14.4	11.6	9.2	8.4	8.7	8.0	9.2	8.4	7.6	8.3
1943	7.5	6.0	7.4	5.9	6.1	6.4	4.9	4.5	4.2	4.0	9.6	6.0
1944	5.3	6.8	6.6	5.1	5.3	6.6	4.9	6.1	6.6	8.5	7.8	7.6
1945	9.3	8.4	10.1	9.8	10.3	12.7	11.2	14.5	20.1	27.6	26.2	21.9
1946	30.3	28.1	31.8	36.6	35.1	36.7	45.7	57.4	57.3	73.9	72.3	74.2
1947	78.0	75.8	90.7	92.5	94.8	101.8	91.0	117.3	122.2	124.4	78.0	54.9
1948	38.9	24.2	18.2	21.7	25.4	27.9	30.8	50.1	56.2	59.4	39.7	27.1
1949	26.2	23.9	25.9	25.5	34.9	46.9	51.6	74.1	94.8	97.0	60.3	40.9

Source: *Survey of Current Business*, Statistical Supplement for 1938 covering 1934–37; for 1942 covering 1938–41; for 1947 covering 1942–46; for 1949 covering 1947–48. March 1950 issue for 1949.

PROBLEM 74. (Continued)

TABLE 57

SEASONAL MOVEMENT OF OIL BURNER SHIPMENTS, UNITED STATES
1934–1941

	Index		*Index*
January	56.5	July	93.8
February	54.8	August	134.9
March	65.5	September	206.9
April	72.9	October	185.6
May	84.1	November	92.2
June	89.6	December	63.2

Source: Computed from data in Table 56.

Questions

1. Do oil burner shipments in the year 1949 revert to their prewar seasonal pattern? Discuss.
2. Is there enough postwar experience on the shipments of oil burners to prepare a reliable postwar seasonal index for this series? If your answer is "no," why, presumably, would our oil burner manufacturer desire your estimate of the new seasonal pattern?
3. Is there any evidence that the business cycle has an influence on the level of oil burner shipments? How do you know?

Adjustment for Trend

PROBLEM 75. REMOVAL OF TREND FROM ANNUAL TOTALS OF PORTLAND CEMENT PRODUCTION

Instructions

1. Take the trend you have calculated by the least squares method, Problem 67, and enter it in Table 58, column *b*, if you have not already done so.
2. Remove the trend from the original data (column *a*) and enter the result in column *c*.
3. Plot the resulting cycle curve. Number your chart, Chart 21.

Questions

1. The original data, column *a* of Table 58, reflect the influence of trend, cyclical and erratic forces which bear upon the cement industry during the period subject to analysis. Which of these three influences have we attempted to remove from the original data by adjustments made in Table 58?
2. What do the ratios in column *c* of Table 58 represent?
3. What does the 100 per cent line in your chart mean? Or in other words, what is the concept of normal when annual totals are used?
4. How might one proceed to remove the erratic factors from the final curve you have plotted in Chart 21?
5. Write a brief paragraph explaining the cyclical swings in Portland Cement Production in recent years.
6. If the *b*-value in your trend expression had been larger what change would have been made in the appearance of your cycle curve?

PROBLEM 75. (Continued)

TABLE 58

ANNUAL PRODUCTION OF PORTLAND CEMENT, UNITED STATES
1910–1949
(In Thousands of Barrels)

Year	*Portland Cement (000 bbls.)*	*Ordinates of Trend*	*Production Adjusted for Trend a/b*
	a	*b*	*c*
1910	76,550		
1911	78,529		
1912	82,438		
1913	92,097		
1914	88,230		
1915	85,915		
1916	91,521		
1917	92,814		
1918	71,082		
1919	80,778		
1920	100,023		
1921	98,842		
1922	114,790		
1923	137,460		
1924	149,358		
1925	161,659		
1926	164,530		
1927	173,207		
1928	176,299		
1929	170,646		
1930	160,908		

PROBLEM 75. (Continued)

TABLE 58 (Continued)

ANNUAL PRODUCTION OF PORTLAND CEMENT, UNITED STATES
1910–1949
(In Thousand of Barrels)

Year	*Portland Cement (000 bbls.)*	*Ordinates of Trend*	*Production Adjusted for Trend a/b*
	a	*b*	*c*
1931	124,572		
1932	76,512		
1933	63,372		
1934	77,688		
1935	76,476		
1936	112,368		
1937	116,484		
1938	105,552		
1939	121,824		
1940	130,296		
1941	164,004		
1942	182,760		
1943	133,488		
1944	90,840		
1945	102,816		
1946	163,800		
1947	186,528		
1948	205,428		
1949	209,831		
Totals			

Source: Data for the period 1910–1934, *Statistical Abstract of the United States, 1940;* p. 798: for 1935–1948, *Survey of Current Business*, Statistical Supplement, 1949, p. 182, (Monthly average×12): for 1949, *Survey of Current Business*, February, 1950, p. S-18.

PROBLEM 76. REMOVAL OF TREND INFLUENCE FROM ANNUAL TOTALS OF PIG IRON PRODUCTION

This problem illustrates how trend may be removed from annual production of pig iron and how a cycle curve may be approximated.

Instructions

1. Take the trend you have calculated by the method of least squares, Problem 69, and enter it in Table 59, Column *b* if you have not already done so.
2. Remove the trend from the original data by dividing out the trend.
3. Plot the data of column *c* and label it, Chart 22.

Questions

1. As a result of the calculations made in this problem and in Problem 69, do you think you have produced a "cycle" curve? If yes, explain how you obtained this result. If not, what do the data charted in Chart 22 represent? Explain.
2. Suppose, in computing the trend in Problem 69, you had fitted it to a somewhat different period and, as a result, your *b*-value was smaller than the one you actually used. What difference would this have made in the appearance of the curve plotted in your Chart 22?
3. What method might be used to smooth out irregular influences remaining in your plotted data?

PROBLEM 76. (Continued)

TABLE 59

ANNUAL PRODUCTION OF PIG IRON IN THE UNITED STATES
1919–1949

Year	*Production of Pig Iron (Thousands of long tons)*	*Ordinates of Trend*	*Production Adjusted for Trend a/b*
	a	*b*	*c*
1919	30,588		
1920	36,420		
1921	16,548		
1922	26,880		
1923	40,056		
1924	31,104		
1925	36,408		
1926	39,072		
1927	36,228		
1928	37,836		
1929	42,288		
1930	31,404		
1931	18,276		
1932	8,688		
1933	13,212		
1934	15,912		
1935	21,010		
1936	30,621		
1937	36,611		
1938	18,782		
1939	31,532		
1940	41,914		
1941	49,918		
1942	53,561		
1943	55,157		
1944	55,307		
1945	48,364		
1946	40,521		
1947	52,864		
1948	54,332		
1949	48,399		

Source: Prior to 1942, these data were compiled by *The Iron Age*, subsequently by *American Iron and Steel Institute*. Data are substantially comparable. Taken from, *Survey of Current Business*, 1940 Supplement, p. 130 for the period 1919–1934 (Monthly averages×12); *Ibid.*, Statistical Supplement, 1949, p. 157 for the period 1935–1948 (converted from short tons); *ibid.*, February, 1950, p. S-32 for 1949.

PROBLEM 77. REMOVAL OF TREND INFLUENCE FROM ANNUAL TOTALS

Instructions

1. Transcribe the trend you have computed, Problem 70, in Table 60 if you have not already done so.

2. Adjust the original data for trend influence and record the resulting ratios in column *c* of Table 60.

3. Construct a chart of the adjusted data and number it, Chart 23.

Questions

1. To what types of economic forces are the variations shown in your Chart 23 attributable? Explain how your calculations in this problem and Problem 70 have produced this result.

2. What is the meaning of the 100 per cent line shown in Chart 23?

3. If your trend, computed in Problem 70, had had a steeper slope, how would the curve shown in Chart 23 have been changed?

4. Can you detect the influence of strikes, cyclical swings and wartime expansion in your Chart 23? Explain. Can you detect the influence of seasonal variations and of trend in Chart 23? Explain.

5. What is the purpose of attempting to isolate variables in economic and business research?

PROBLEM 77. (Continued)

TABLE 60

ANNUAL ANTHRACITE COAL PRODUCTION IN PENNSYLVANIA, 1916–1949

Year	*Anthracite Coal (000 of Short Tons)*	*Ordinates of Trend*	*Production Adjusted for Trend a/b*
	a	*b*	*c*
1916	87,578		
1917	99,612		
1918	98,826		
1919	88,092		
1920	89,598		
1921	90,473		
1922	54,683		
1923	93,339		
1924	87,927		
1925	61,817		
1926	84,437		
1927	80,096		
1928	75,348		
1929	73,828		
1930	69,385		
1931	59,646		
1932	49,855		
1933	49,541		
1934	57,168		
1935	52,164		
1936	54,576		
1937	51,852		
1938	46,104		
1939	51,492		
1940	51,480		
1941	56,364		
1942	60,324		
1943	60,648		
1944	63,696		
1945	54,936		
1946	60,504		
1947	57,192		
1948	57,048		
1949	42,664		

Source: *Statistical Abstract of the United States*, 1940, p. 784 for years 1916–1934; *Survey of Current Business*, Statistical Supplement for 1949, p. 168 for the years 1935 through 1948; *ibid*, February 1950, p. S-34, for 1949 data.

PROBLEM 78. TIME SERIES ANALYSIS APPLIED TO MONTHLY VALUES, PORTLAND CEMENT PRODUCTION, UNITED STATES

When the analysis of a time series is concerned with annual totals, as in the previous problems, seasonal factors do not enter to complicate the calculations. Often, however, such time series analysis must be performed on monthly data. This is a somewhat more complicated process.

In this problem, we must combine the results of trend and seasonal variations in cement production. In order to solve this problem we must have (*a*) a trend in monthly rather than annual values and (*b*) an index of seasonal variation in the production of cement.

When monthly data are to be analyzed, the trend is usually calculated for annual totals, as in Problem 67. The annual total trend must then be reduced to monthly values. (See instructions on pages 521–526 of the text.)

Instructions

1. The trend you will use in this problem is from Problem 67.
2. In Table 62 is shown monthly production of Portland cement for the years 1947, 1948, and 1949. Calculate and enter in column *b* of Table 62 the monthly ordinates of trend.
3. Enter the values of the seasonal index shown in Table 61 in column *c* of Table 62.
4. Compute the statistical normals by months for 1947, 1948, and 1949 and enter in column *d*.
5. Reduce actual monthly production to percentages of normal and enter the results in column *e*.
6. Plot monthly Portland cement production for 1947, 1948, and 1949 as it deviates from normal, and number your chart, Chart 24.
7. Examine Chart 24 critically. Do you think you have succeeded in removing the seasonal influence from the original data? Explain.

Questions

1. What is the definition of normal (meaning of 100 on your chart) when monthly data are used? How does this differ from the definition of normal for annual data?
2. After adjustments have been made for trend and seasonal variation, what was the position of the industry in July 1949 as contrasted with July 1947? What is the percentage change?
3. Is the change from July 1947 to July 1949 due to seasonal or trend factors? If neither, what type of fluctuations are they and how do you know?
4. Do you think you have "overcompensated" or "undercompensated" for the seasonal variation? How would you tell by study of your Chart 24? Explain.

TABLE 61

SEASONAL INDEX OF PORTLAND CEMENT PRODUCTION, UNITED STATES

	Index		*Index*
January	81.35	July	110.40
February	75.89	August	112.90
March	81.80	September	115.83
April	94.89	October	116.13
May	105.00	November	107.90
June	107.06	December	90.85

PROBLEM 78. (Continued)

TABLE 62

MONTHLY PORTLAND CEMENT PRODUCTION, CALCULATION OF NORMAL AND ISOLATION OF CYCLE 1947, 1948, AND 1949

Y_c = _____ + _____ X; X = 1 month; origin at _____, 19__

	Portland Cement Production (000 barrels)	*Ordinates of Monthly Trend*	*Seasonal Index*	*Normal Production*	*Cyclical and Erratic Influence*
	a	*b*	*c*	*d*	*e*
1947					
Jan.	13,406				
Feb.	12,618				
Mar.	14,205				
Apr.	14,566				
May	13,389				
June	15,971				
July	16,342				
Aug.	17,480				
Sept.	17,319				
Oct.	18,300				
Nov.	16,814				
Dec.	16,123				
1948					
Jan.	14,541				
Feb.	13,347				
Mar.	14,502				
Apr.	16,041				
May	17,740				
June	17,757				
July	18,721				
Aug.	18,961				
Sept.	18,605				
Oct.	19,349				
Nov.	18,435				
Dec.	17,425				

PROBLEM 78. (Continued)

TABLE 62 (Continued)

MONTHLY PORTLAND CEMENT PRODUCTION, CALCULATION OF NORMAL AND ISOLATION OF CYCLE 1947, 1948, AND 1949

Y_c = _____ + _____ X; X = 1 month; origin at _____, 19__

	Portland Cement Production (000 barrels)	*Ordinates of Monthly Trend*	*Seasonal Index*	*Normal Production*	*Cyclical and Erratic Influence*
	a	*b*	*c*	*d*	*e*
1949					
Jan.	15,261				
Feb.	13,751				
Mar.	15,439				
Apr.	17,682				
May	18,622				
June	18,279				
July	18,856				
Aug.	18,715				
Sept.	19,181				
Oct.	19,070				
Nov.	18,040				
Dec.	16,936				

Source: 1947–1948, *Survey of Current Business*, Statistical Supplement, 1949, p. 182; 1949, *Survey of Current Business*, March 1950, p. S-38.

PROBLEM 79. THE CALCULATION OF NORMAL FOR MONTHLY DATA. PIG IRON PRODUCTION

In computing a statistical normal for pig iron production in the postwar period, the analyst is confronted with special and possibly insolvable problems. As noted in the text, a statistical normal for monthly data is defined as $T \times S$. The trend computed in Problem 69 may be reduced to monthly magnitudes and used in estimating normal. But there is no satisfactory seasonal index for use in completing the estimates of normal for recent years.

In Table 63 is shown the prewar seasonal pattern. A glance at Chart 9 will indicate, however, that this pattern did not persist during the war years and, if it is present in the postwar period, it is entirely obscured by many erratic forces. During the war, of course, the industry operated at capacity or near capacity. A people engaged in an industrial war cannot permit usual seasonal let-downs. In the postwar period there have been strikes in both coal and steel which have left their imprint, and the market has been such as to take all products which the industry is capable of producing.

What then does one do in estimating monthly values which are to be considered "normal" for the period? This is the problem to which the student must find an answer which he considers defensible if not entirely satisfactory.

PROBLEM 79. (Continued)

Instructions

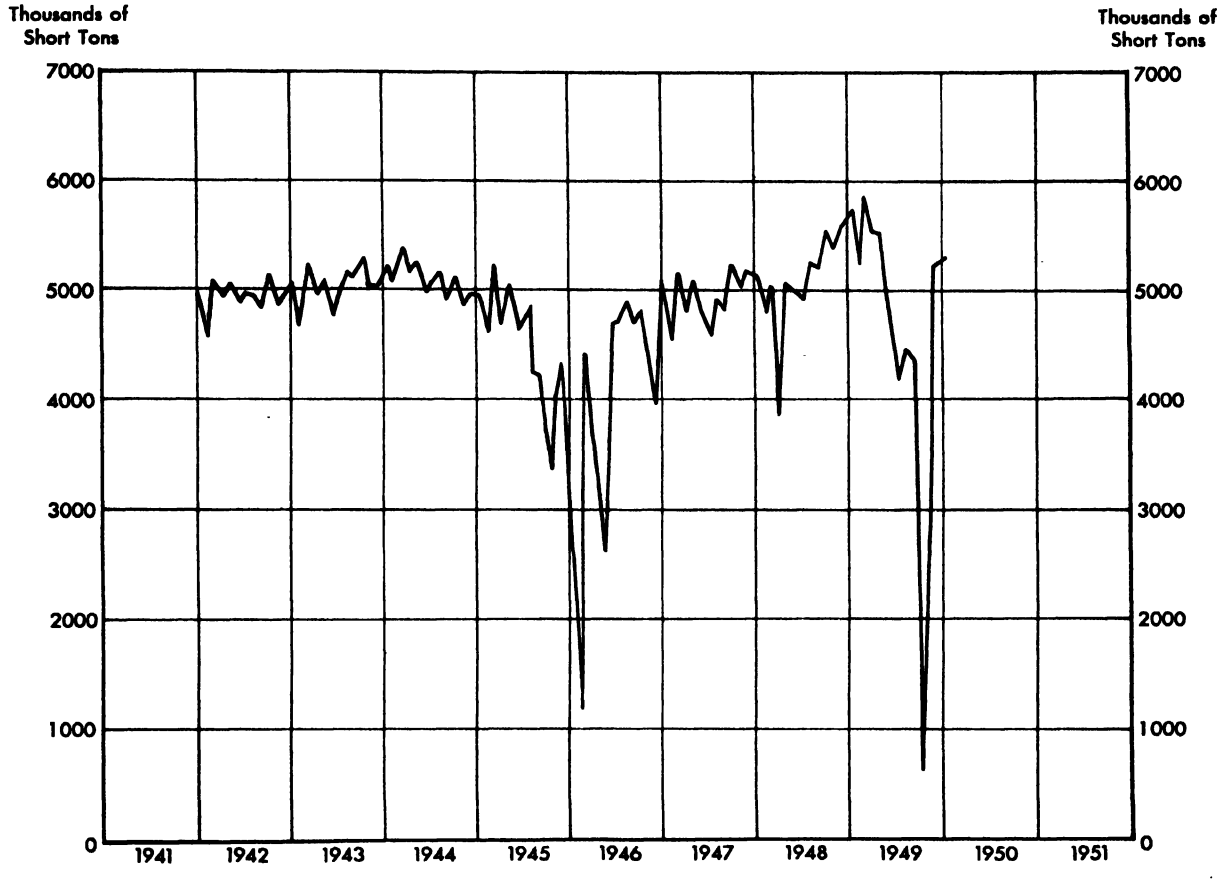
1. Reduce the trend computed in Problem 69 to monthly magnitudes.

2. Compute the monthly trend values for the period shown in Table 64 and record your results in column *b*, Table 64.

3. Decide what, if anything, to do about the seasonal adjustment and complete the calculations for column *d*, Table 64. Justify your decision concerning the seasonal adjustment.

CHART 9

MONTHLY PRODUCTION OF PIG IRON
UNITED STATES, 1942–1949



Source: Table 64.

4. Divide the data of column *a* by your estimates of monthly normal and enter the results in column *e*, Table 64.

5. Plot monthly pig iron production in 1947–1949 as it deviates from your estimate of normal. Label this Chart 25.

6. Review your efforts to compute a statistical normal in this case and evaluate the results.

PROBLEM 79. (Continued)

TABLE 63

PREWAR SEASONAL MOVEMENT OF PIG IRON PRODUCTION
UNITED STATES

January	101.0	July	99.0
February	94.2	August	99.0
March	108.3	September	94.1
April	108.1	October	98.2
May	108.1	November	94.5
June	100.9	December	94.9

Questions

1. What is the definition of normal (the meaning of 100 on your chart) when monthly data are used? How does this differ from the definition of normal for annual data?

2. After adjustments have been made for trend and seasonal variation, what was the position of the industry in December 1949, as contrasted with December 1947? What is the percentage change?

3. Is the change from December 1947 to December 1949 shown as deviations from normal due to seasonal or trend factors? If not, what type of fluctuation is it and how do you know?

TABLE 64

MONTHLY PIG IRON PRODUCTION, CALCULATION OF NORMAL AND ISOLATION OF CYCLE
1947–1949

$Y_c =$ _____ + _____ X; X = 1 month; origin at _____, 19__

	Pig Iron Production (000 long tons)	*Ordinates of Monthly Trend*	*Seasonal Index*	*Normal Production*	*Cyclical and Erratic Influence*
	a	*b*	*c*	*d*	*e*
1947					
Jan.	5087				
Feb.	4550				
Mar.	5123				
Apr.	4830				
May	5081				
June	4810				
July	4585				
Aug.	4917				
Sept.	4801				
Oct.	5228				
Nov.	5015				
Dec.	5177				

PROBLEM 79. (Continued)

TABLE 64 (Continued)

MONTHLY PIG IRON PRODUCTION, CALCULATION OF NORMAL AND ISOLATION OF CYCLE
1947–1949

Y_c = _____ + _____ X; X = 1 month; origin at _____, 19__

	Pig Iron Production (000 long tons)	*Ordinates of Monthly Trend*	*Seasonal Index*	*Normal Production*	*Cyclical and Erratic Influence*
	a	*b*	*c*	*d*	*e*
1948					
Jan.	5128				
Feb.	4870				
Mar.	5020				
Apr.	3840				
May	5077				
June	4991				
July	4900				
Aug.	5255				
Sept.	5208				
Oct.	5520				
Nov.	5399				
Dec.	5595				
1949					
Jan.	5732				
Feb.	5223				
Mar.	5820				
Apr.	5531				
May	5517				
June	4819				
July	4173				
Aug.	4477				
Sept.	4350				
Oct.	612				
Nov.	2722				
Dec.	5231				

Source: *Survey of Current Business*, Statistical Supplement, 1949, for years 1947–1948, p. 158; *ibid*, February 1950, for 1949, p. S-32.

PROBLEM 80. TIME SERIES ANALYSIS APPLIED TO MONTHLY VALUES, ANTHRACITE COAL PRODUCTION

This problem is to be used in conjunction with Problem 70 where the trend in anthracite coal production was analyzed. Now it is desired to reduce the trend in annual magnitudes to monthly magnitudes, make an adjustment for the expected seasonal variation, and compute a "statistical normal" in monthly values.

Instructions

1. The trend which you will work with in this problem is the one you computed in Problem 70. Reduce it to monthly magnitudes.
2. In Table 66 is shown the monthly production of anthracite for the years 1946, 1947, and 1948. Calculate and enter in column *b* the monthly ordinates of trend.
3. Enter the values of the seasonal index shown in Table 65 in column *c* of Table 66.
4. Compute the statistical normal for monthly anthracite coal production by months for the years 1946, 1947, and 1948, and enter in column *d*.
5. Reduce the actual monthly production to percentages of normal and enter the results in column *e*.
6. Plot monthly anthracite coal production in 1946, 1947, and 1948 as it deviates from normal and number this Chart 26. Examine this chart critically to see whether you have over- or under-compensated for the seasonal variation.

Questions

1. What is the result of your analysis undertaken in Instruction 6? How can you tell whether you have under or over compensated for the seasonal effects? Explain.
2. What influences account for the variations shown in your Chart 26? Can the data be further smoothed or simplified better to reveal cyclical variations? Explain.
3. What does the 100 per cent line represent in your Chart 26?
4. How does the calculation made in Problem 70 affect the appearance of the curve, Chart 26?
5. Write a brief paragraph describing the condition of this industry with respect to its expectation during the months under analysis.

TABLE 65

TYPICAL SEASONAL VARIATION PRODUCTION OF ANTHRACITE COAL IN PENNSYLVANIA

January	123.6	July	75.3
February	103.1	August	79.9
March	96.4	September	94.6
April	101.7	October	107.8
May	109.8	November	96.8
June	100.8	December	110.2

PROBLEM 80. (Continued)

TABLE 66

MONTHLY ANTHRACITE PRODUCTION, CALCULATION OF NORMAL AND ISOLATION OF CYCLE AND ERRATIC VARIATIONS, 1946 AND 1948

Y_c = _____ + _____ X; X = 1 month; origin at _____, 19__

	Monthly Anthracite Production (000 short tons)	*Ordinates of Monthly Trend*	*Anthracite Seasonal Index*	*Normal Production*	*Cyclical and Erratic Influence*
	a	*b*	*c*	*d*	*e*
1946					
Jan.	4968				
Feb.	4774				
Mar.	5476				
Apr.	5069				
May	5453				
June	3625				
July	5248				
Aug.	5428				
Sept.	5033				
Oct.	5393				
Nov.	4975				
Dec.	5065				
1947					
Jan.	5172				
Feb.	4254				
Mar.	4984				
Apr.	4293				
May	4564				
June	4624				
July	4098				
Aug.	5011				
Sept.	5158				
Oct.	5524				
Nov.	4629				
Dec.	4879				

PROBLEM 80. (Continued)

TABLE 66 (Continued)

MONTHLY ANTHRACITE PRODUCTION, CALCULATION OF NORMAL AND ISOLATION OF CYCLE AND ERRATIC VARIATIONS, 1946 AND 1948

Y_c = _____ + _____ X; X = 1 month; origin at _____, 19__

	Monthly Anthracite Production (000 short tons)	*Ordinates of Monthly Trend*	*Anthracite Seasonal Index*	*Normal Production*	*Cyclical and Erratic Influence*
	a	*b*	*c*	*d*	*e*
1948					
Jan.	4921				
Feb.	4675				
Mar.	4928				
Apr.	4438				
May	4867				
June	4590				
July	4365				
Aug.	5121				
Sept.	5007				
Oct.	4961				
Nov.	4680				
Dec.	4499				

Source: *Survey of Current Business*, 1949 Statistical Supplement, p. 168.

PROBLEM 81. USE OF TREND AND SEASONAL ESTIMATES IN ANALYZING BUSINESS DATA

TABLE 67

SALES OF ACME CORPORATION AND COMPUTED TREND, 1949

	Sales	*Trend*
January	$120,000	$100,000
February	130,000	100,500
March	142,000	101,000
April	146,000	101,500
May	119,000	102,000
June	100,000	102,500
July	140,000	103,000
August	156,000	103,500
September	160,000	104,000
October	145,000	104,500
November	160,000	105,000
December	180,000	105,500
Total	$1,698,000	$1,233,000

PROBLEM 81. (Continued)

TABLE 68

TYPICAL SEASONAL VARIATION IN SALES, ACME CORPORATION

	Index		*Index*
January	80	July	90
February	90	August	100
March	110	September	110
April	120	October	100
May	90	November	110
June	70	December	130

Source: Adapted from *Mid-Year Examination in Laboratory Method*, Business Statistics 1, Harvard Business School.

Questions

1. Did the business of the Acme Corporation improve or become worse between January and April 1949? between September and December 1949? State the percentage change in each case.
2. If business is "normal" in March 1953, what will the sales volume be for that month?
3. If the officials expect business in 1951 to be 20 per cent above normal, what sales volume will they anticipate?

SECTION X

Functional Relations and Correlation Analysis

1. Functional Relations and Estimates (Chapter XVI of text), Problems 82, 83, 85
2. Correlation (Chapter XVII of text), Problems 84, 86, 87)
3 Summary and Review, Problems 88, 89, 90

PROBLEM 82. INTERPRETATION OF SCATTER DIAGRAMS

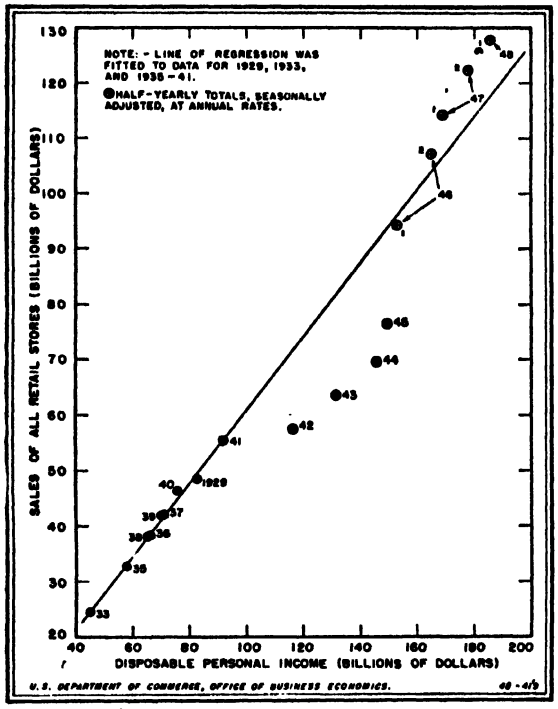
The scatter diagrams reproduced below come from the October, 1948, *Survey of Current Business*, published monthly by the U. S. Department of Commerce. The study of which these exhibits are a part is entitled "Retail Sales and Consumer Income" and correlation technique is used to analyze the movement of sales and to forecast sales of certain classes of goods.

Questions

1. Refer to Exhibit 4. Were all of the values shown in the scatter diagram used in computing the level and slope of the line of regression? What justification, if any, can you make for the method used?
2. How do you explain the behavior of the values in the period 1942–1945 (Exhibit 4)?
3. In 1949, disposable personal income is estimated to be approximately $191 billion. On the basis of the relationships shown in Exhibit 4, what is your estimate of total sales of retail stores at that level of income? Actual sales of retail stores in 1949 were about $128 billion. Account for any differences between this value and your estimate.
4. In general, do the American people seem to be spending more or less of each dollar of additional disposable personal income in retail stores than they did in the prewar period? Explain. If prewar relations between disposable income and sales of retail stores are reestablished, would you expect sales of retail stores to be lower than they have been in recent years for similar levels of income? Explain.
5. How, if at all, do you think the following conditions have influenced the scatter shown in Exhibit 4?
   a. The increase of population 1933 to 1948.
   b. The rise in commodity prices 1933 to 1948.
   c. Rent controls imposed by Federal action, 1942 to 1948.
   d. The relatively more rapid rise in the prices of agricultural produce.
6. Refer to Exhibit 5 and make the same analysis for it as was made under 1 above.
7. Relative to expectation, as revealed in the regression lines, which category of goods, nondurable or durable, declined most during the war years? How do you explain this? (Exhibit 5.)
8. Which category of sales, nondurable or durable, seems to have increased most, relative to expectation, in the postwar period? How do you explain this?
9. Which category would you expect to get an increased amount of the income dollar in 1949, the nondurable or the durable goods? Why? (Refer to recent copies of the *Survey of Current Business* to learn whether your expectations were realized.)
10. Point out the limitations, or opportunities for error, in this type of statistical-economic analysis and estimating.

EXHIBIT 4

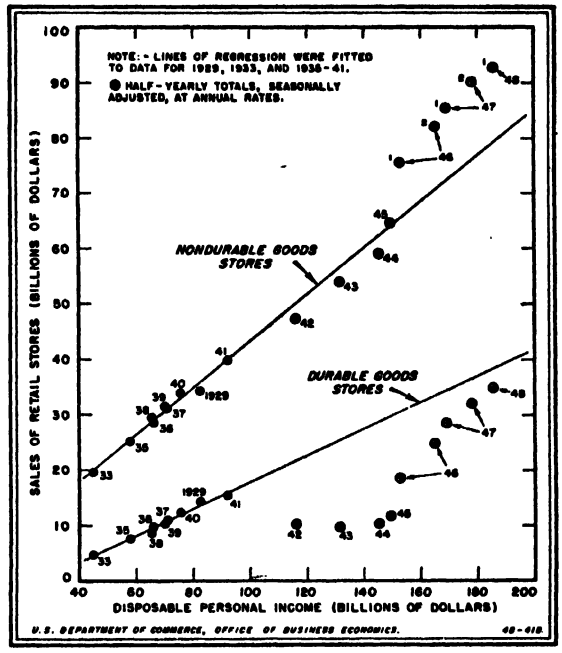
RELATIONSHIP BETWEEN SALES OF ALL RETAIL STORES AND DISPOSABLE INCOME, UNITED STATES SELECTED YEARS, 1929–1948



Source: *Survey of Current Business*, U. S. Department of Commerce, October 1948.

EXHIBIT 5

RELATIONSHIP BETWEEN SALES OF DURABLE AND NONDURABLE GOODS, RETAIL STORES AND DISPOSABLE INCOME UNITED STATES, SELECTED YEARS 1929–1948



Source: *Survey of Current Business*, U. S. Department of Commerce, October 1948.

PROBLEM 83. REGRESSION LINES AND STATISTICAL ESTIMATES

The method of analysis used in the previous problem may be applied to the sales of Sears Roebuck and Company. Sales of this Company, 1931–1949, and disposable income, for the same period, are shown in Table 69 together with certain values needed in computing the constants in the regression equation.

Instructions

1. Consider whether an arithmetic or semi-logarithmic scale should be used in making a line chart of these data and prepare the Chart. Number it Chart 26.
2. Make a scatter diagram for the data 1931–1941. This is Chart 27.
3. Compute the values of the regression equation.
4. Plot the regression line on the scatter diagram.
5. Explain the meaning of the *b*-value in the regression statement. Compute the "typical" increase in Sears Roebuck and Company sales which has accompanied an increase of 1 billion dollars in disposable personal income during the period 1931–1941.
6. Compute the standard error of estimate.

PROBLEM 83. (Continued)

TABLE 69

SEARS ROEBUCK AND COMPANY SALES DISPOSABLE PERSONAL INCOME UNITED STATES 1931–1949

Year	*Sears Roebuck Sales Millions of Dollars* Y	*Disposable Personal Inc. (Billions)* X	XY	Y^2	X^2
1931	$ 347	$63	21,861	120,409	3,669
1932	280	48	13,440	78,400	2,304
1933	285	45	12,825	81,225	2,025
1934	337	52	17,524	113,569	2,704
1935	414	58	24,012	171,396	3,364
1936	521	66	34,386	271,441	4,356
1937	574	71	40,754	329,476	5,041
1938	533	66	35,178	284,089	4,356
1939	651	70	45,570	423,801	4,900
1940	740	76	56,240	547,600	5,776
1941	956	92	87,952	913,936	8,464
1942	918	117			
1943	873	132			
1944	1,006	147			
1945	1,058	151			
1946	1,667	158			
1947	2,090	172			
1948	2,418	191			
1949	2,274	191			
Totals 1931–41	5,638	707			

Source: Annual Supplement *Survey of Current Business*, 1931–34 data from 1940 issue; 1935–48 data from Statistical Supplement; *ibid.*, 1949 data, February 1950 issue *Survey of Current Business*.

7. Estimate Sears Roebuck and Company sales when disposable personal income is $70 billion as in 1939. Compare your estimate with the actual sales in 1939. How do you explain any difference which may appear.

8. Assume that the same relation between increased sales and increases in disposable personal income as existed 1931–1941 is maintained at much higher levels of income and estimate sales when disposable income is $172 billions (1947); $191 billions (1948 and 1949). Estimate also, sales for 1951 by assuming a value for disposable income which you think reasonable for 1951.

Questions

1. Defend your choice of scale used in the line chart.

2. Can you justify the use of the period 1931–1941 and the exclusion of war and postwar data in this correlation analysis? Explain in some detail by reference to your line chart and the economic history it reveals.

3. Between 1931 and 1941 were these two series affected similarly by the "cycles" in business activity? by the trend? by the changing value of money? What difference do the answers to these questions make in the correlation time series data?

4. (a) What statements are you able to make as to the error in your estimate made in instruc-

PROBLEM 83. (Continued)

tion 7? (b) Can you make any similar statements concerning the estimates you made under instruction 8 for the postwar years? Explain.

5. Relative to prewar relations, is Sears Roebuck and Company doing as well as would be expected in recent years? Explain. In answering this question consider critically the assumption made in Instruction 8.

PROBLEM 84. THE MEANING OF CORRELATION

Instructions

1. Refer to the data of Table 69, previous problem, and compute the standard deviation of Y.
2. Compute the coefficient of determination. (You have Sy from the previous problem.)
3. Compute the coefficient of correlation.

Questions

1. What per cent of the variation in Sears Roebuck and Company sales seems to be explained by changes in disposable personal income? What is the name given to this measure?

2. If the standard deviation of Y and the standard error of Y are identical in value, what would the coefficient of correlation be? How would you explain this result?

3. (*a*) Should these data have been deflated before the correlation analysis was made? Explain. (*b*) Do you think the correlation coefficient would have been even higher than the result obtained had the data been deflated? lower? Explain.

4. Do you think the correlation found in this problem is "real" or "spurious"? Explain.

5. Do you think it would have been better to use "Cash Farm Income" in the U. S. as the independent variable in the analysis of Sears Roebuck and Company sales? Explain.

6. When samples are correlated, what difference in interpreting "r" does the size of sample make?

7. Would you say you have been working with a random sample of sales and income in this problem? Discuss.

PROBLEM 85. U. S. DEMAND FOR LUBRICATING OILS AND INDUSTRIAL PRODUCTION

The U. S. domestic demand for lubricating oils tends to rise and fall with the level of industrial activity in the United States. A forecast of the level of lubricating oil demand for 1952 is needed and this relationship can be used to make an estimate for 1952. Data for our calculation appear in Table 70.

Instructions

1. Plot the series on industrial production and demand for lubricating oils first as a line chart and then as a scatter diagram. Label your charts, Chart 28 and Chart 29, respectively.

2. Compute the average relationship over the period 1919–1949 between the Federal Reserve Board Index of Industrial Production (1935–39 = 100) and the domestic demand for lubricating oils. Draw this regression line on Chart 29.

3. Compute the standard error of estimate.

4. Estimate the level of domestic demand for lubricating oils for 1952 under the assumption that the FRB index for that year is at 190.

Questions

1. The demand for lubricating oils comprises the demand for motor oil for passenger cars, trucks and aircraft, oil for farm tractors and for a vast number of industrial uses. Does it seem reason-

PROBLEM 85. (Continued)

TABLE 70

U. S. DEMAND FOR LUBRICATING OILS AND THE FEDERAL RESERVE BOARD INDEX OF INDUSTRIAL PRODUCTION (1935–39 = 100)
1919–1949

Year	*Demand for Lubricating Oils (Millions of Bbls.)*	*FRB Index of Industrial Production (1935–39 = 100)*	X^2	Y^2	XY
1919	13.6	72	5,184	185.0	979.2
1920	14.7	75	5,625	216.1	1,102.5
1921	12.0	58	3,364	144.0	696.0
1922	15.5	73	5,329	240.2	1,131.5
1923	17.6	88	7,744	309.8	1,548.8
1924	18.1	82	6,724	327.6	1,484.2
1925	20.6	90	8,100	424.4	1,854.0
1926	22.6	96	9,216	510.8	2,169.6
1927	21.7	95	9,025	470.9	2,061.5
1928	23.2	99	9,801	538.2	2,296.8
1929	23.6	110	12,100	557.0	2,596.0
1930	21.6	91	8,281	466.6	1,965.6
1931	19.9	75	5,625	396.0	1,492.5
1932	16.6	58	3,364	275.6	962.8
1933	17.2	69	4,761	295.8	1,186.8
1934	18.5	75	5,625	342.2	1,387.5
1935	19.7	87	7,569	388.1	1,713.9
1936	22.3	103	10,609	497.3	2,296.9
1937	23.3	113	12,769	542.9	2,632.9
1938	21.2	89	7,921	449.4	1,886.8
1939	23.7	109	11,881	561.7	2,583.3
1940	24.7	125	15,625	610.1	3,087.5
1941	30.3	162	26,244	918.1	4,908.6
1942	29.1	199	39,601	846.8	5,790.9
1943	31.4	239	57,121	986.0	7,504.6
1944	32.4	235	55,225	1,049.8	7,614.0
1945	35.3	203	41,209	1,246.1	7,165.9
1946	34.9	170	28,900	1,218.0	5,933.0
1947	36.5	187	34,969	1,332.2	6,825.5
1948	36.0	192	36,864	1,296.0	6,912.0
1949	33.0	176	30,976	1,089.0	5,808.0
Totals	730.8	3,695	527,351	18,731.7	97,579.1

Source: FRB Index of Industrial Production, *Federal Reserve Bulletin*, March 1950; Lubricating Oil Demand, *Statistical Bulletin*, American Petroleum Institute and Bureau of Mines, Monthly Petroleum Statement.

PROBLEM 85. (Continued)

able to you that each of these demands would correlate to the same degree with the FRB Index? Discuss.

2. What statements, if any, can you make about the probability that the actual 1952 demand will be within the range of your estimate $\pm Sy$?

3. This problem involves correlation of time series data and no attempt has been made to eliminate the trend, cycle or episodic variations. Discuss the limitations of the methodology and if you can, justify the procedure used.

4. In certain years the "typical" relation found between the series does not seem to hold. What factors, in your opinion, account for such departures from the usual relationship.

The Measurement of Correlation

PROBLEM 86. PER CAPITA INCOME AND PER CAPITA SALES OF LIFE INSURANCE

A regional sales manager of a life insurance firm is interested in measuring the recent relationship between income in various states and purchases of ordinary life insurance. You are asked to prepare the analysis.

Data for 1948 are available on insurance purchases and income payments by states. In order to remove variations attributable to the size of the state, each series is put on a per capita basis, and the tabulations, as presented in Table 71, are prepared.

Instructions

1. As a basis for your report, calculate the coefficient of correlation (using the standard error of estimate and the standard deviation of the Y-series) for per capita income by states and per capita purchases of ordinary life insurance by states. Use the following formula for calculation of the standard deviation of the Y-series:

$$SD_y^2 = \left(\frac{\Sigma Y^2}{N}\right) - \left(\frac{\Sigma Y}{N}\right)^2$$

2. Calculate the following measures also:
   a. Regression equation.
   b. Coefficient of determination.
   c. Coefficient of non-determination.
   d. Coefficient of alienation.

3. Write out in terms of income and life insurance sales, the meaning of each of the measures you have calculated.

Questions

1. How might the measures of correlation you have computed in this problem be used by the insurance company sales manager?

2. It is possible that the coefficient of correlation you obtained appeared as a result of chance errors of sampling and that the correlation between the variables is actually zero. Is that probable? How do you know?

3. Estimate per capita sales of life insurance in a state with per capita income payments of $1,500.

4. If the state for which the estimate was made, in 3 above, had a population of 3.5 million, what is your estimate of total life insurance sales in that state?

PROBLEM 86. (Continued)

TABLE 71

PER CAPITA SALES OF LIFE INSURANCE AND PER CAPITA INCOME PAYMENTS 1948 BY STATES
Calculation of Values Used in Measuring Functional Relation between Variables

State	*Per Capita Income Payments*	*Per Capita Sales of Life Insurance*			
	X ($)	Y ($)	XY	X^2	Y^2
Maine	1,219	77	93,863	1,485,961	5,929
New Hamsphire	1,261	94	118,534	1,590,121	8,836
Vermont	1,229	107	131,503	1,510,441	11,449
Massachusetts	1,509	98	147,882	2,277,081	9,640
Rhode Island	1,564	110	172,040	2,446,096	12,100
Connecticut	1,700	126	214,200	2,890,000	15,876
New York	1,891	126	238,266	3,575,881	15,876
New Jersey	1,605	122	195,810	2,576,025	14,884
Pennsylvania	1,444	105	151,620	2,085,136	11,025
Ohio	1,548	108	167,184	2,396,304	11,664
Indiana	1,403	95	133,285	1,968,409	9,025
Illinois	1,817	121	219,857	3,301,489	14,641
Michigan	1,484	96	142,464	2,202,256	9,216
Wisconsin	1,443	103	148,629	2,082,249	10,609
Minnesota	1,353	99	133,947	1,830,609	9,801
Iowa	1,491	103	153,573	2,223,081	10,609
Missouri	1,356	96	130,176	1,838,736	9,216
North Dakota	1,473	93	136,989	2,169,729	8,649
South Dakota	1,577	103	162,431	2,486,929	10,609
Nebraska	1,473	111	163,503	2,169,729	12,321
Kansas	1,291	102	131,682	1,666,681	10,404
Delaware	1,741	130	226,330	3,031,081	16,900
Maryland	1,546	106	163,876	2,390,116	11,236
Dist. of Columbia	1,691	149	251,959	2,859,481	22,201
Virginia	1,159	91	105,469	1,343,281	8,281
West Virginia	1,133	70	79,310	1,283,689	4,900
North Carolina	930	68	63,240	864,900	4,624
South Carolina	865	54	46,710	748,225	2,916
Georgia	971	73	70,883	942,841	5,329
Florida	1,137	94	106,878	1,292,769	8,836
Kentucky	909	56	50,904	826,281	3,136
Tennessee	955	61	58,255	912,025	3,721
Alabama	891	54	48,114	793,881	2,916
Mississippi	758	38	28,804	574,564	1,444
Arkansas	863	42	36,246	744,769	1,764
Louisiana	1,002	55	55,110	1,004,004	3,025
Oklahoma	1,029	85	87,465	1,058,841	7,225
Texas	1,192	101	120,392	1,420,864	10,201
Montana	1,791	114	204,174	3,207,681	12,996
Idaho	1,252	85	106,420	1,567,504	7,225
Wyoming	1,494	105	156,870	2,232,036	11,025
Colorado	1,429	133	190,057	2,042,041	17,689
New Mexico	1,125	70	78,750	1,265,625	4,900
Arizona	1,168	72	84,096	1,364,224	5,184
Utah	1,231	116	142,796	1,515,361	13,456
Nevada	1,679	104	174,616	2,819,041	10,816
Washington	1,453	98	142,394	2,111,209	9,604
Oregon	1,302	96	124,992	1,695,204	9,216
California	1,651	116	191,516	2,725,801	13,456
Totals	65,478	4,631	6,484,064	91,410,282	466,565

Source: Income payments, *Survey of Current Business*, U. S. Department of Commerce, August, 1949, p. 15; life insurance sales, *Life Insurance Fact Book*, 1949, p. 14; population, *Current Population Report*, Department of Commerce, Bureau of Census, August 5, 1949.

PROBLEM 86. (Continued)

5. What was the reason for using per capita values in this problem rather than actual income and actual sales? Would the correlation have been higher or lower, in your opinion, if actual rather than per capita values had been used? Explain.

6. What per cent of the variation in per capita life insurance sales among the states seems to be explained by variations in income per capita.

Analysis of Functional Relationships

PROBLEM 87. PRICE AS A FUNCTION OF SUPPLY

Early in the usual introductory course in Principles of Economics the student becomes acquainted with the Law of Demand. This law is a statement of a functional relationship. It holds that in a given competitive market at a given time, the price which would be paid varies inversely with supply. It is the objective of this problem, in so far as we are able by elementary methods, to test this proposition.

In Table 72 we have (in column *a*) prices of Louisiana strawberries, as quoted in the Chicago market, and (in column *b*) the daily arrivals at that market of carloads of strawberries over the period April 7 to May 8, 1941. The series for carload arrivals has been smoothed by a five-term moving average centered on the middle term (column *c*). The data have been edited in this way because all shipments are not sold on the day of arrival but are often reserved for favorable terms within the time limits of the commodity's perishability. Further, because arrivals accumulate over the weekend and are recorded as Monday arrivals, the moving average is needed to smooth out these extreme variations. Finally, buyers have information as to the shipments en route so that prices on Tuesday, for instance, are determined in part by strawberries in transit which will not arrive, say, until Thursday. Thus present prices are determined in part by anticipated supply. The moving average, centered on the middle term, takes cognizance of these facts.

On the basis of these data we wish to discover whether there is, as economics teaches, a functional relationship between price and supply, and if so, the general statistical characteristics of that function. Actually, of course, the economic concept of a demand schedule is highly abstract. It purports to explain a schedule of prices which would prevail under given conditions of supply at an instant of time. Yet in the perfectly competitive market at any one time there is but one price. Clearly, any realistic attempt to construct a demand schedule from actual data is involved in a basic difficulty.

Our task is to bridge the gap from a time series, a dynamic situation, to a frequency distribution, in the sphere of the static. In attempting to construct a demand curve we must use time series data to see what happens to price as supply changes. Yet with the passage of time other factors enter. The value of money may change. Fads and fashions change. Further, as has been explained in the text, similar or opposite trends in the two series, similar or opposite seasonals or cycles may cause functional relationships to appear which are in fact spurious. In data collected from the markets of the world the simple conditions assumed in static economic theory are not present.

Anyone attempting to relate the abstract propositions of economic theory to the facts of experience as revealed in quantitative data must be mindful of the difficulties outlined above. With an awareness of these problems, however, we are ready to proceed to the analysis of the data.

Instructions

1. Plot the data in Table 72 as a time series. Number your chart, Chart 30.

2. Decide which is the independent variable and plot the data in a scatter diagram. Number your chart, Chart 31.

PROBLEM 87. (Continued)

TABLE 72

AVERAGE PRICE OF STRAWBERRIES IN CHICAGO MARKET AND CHICAGO ARRIVALS OF STRAWBERRIES BY CARLOAD*

April 7 to May 8, 1941

Date	*Average Price per Crate† First Quality* (*Dollars*) (*a*)	*Arrivals by Carload* (*Number*) (*b*)	*Five-Term Moving Average* (*Arrivals*) (*c*)
Mon. April 7	4.25	3	
Tues. 8	4.30	1	
Wed. 9	4.50	3	2.6
Thu. 10	4.50	3	3.8
Fri. 11	4.68	3	3.6
Mon. 14	3.75	9	3.8
Tues. 15	3.38	0	4.4
Wed. 16	3.50	4	5.8
Thu. 17	3.43	6	9.8
Fri. 18	2.95	10	11.8
Mon. 21	2.15	29	13.8
Tues. 22	2.25	10	15.2
Wed. 23	2.75	14	16.2
Thu. 24	2.58	13	18.4
Fri. 25	2.48	15	18.8
Mon. 28	2.35	40	19.2
Tues. 29	2.43	12	20.0
Wed. 30	2.58	16	22.8
Thu. May 1	2.45	17	25.2
Fri. 2	2.20	29	27.8
Mon. 5	1.95	52	28.4
Tues. 6	1.88	25	29.4
Wed. 7	1.70	10	
Thu. 8	1.70	22	

* The strawberries considered are "Klondikes" from Louisiana and arrive in Chicago almost entirely by rail. The Louisiana strawberry season runs most of its course before berries of other neighboring states enter this market.

† A crate here consists of 24 pints.

Source: *Chicago Fruit and Vegetable Market Reports*, U. S. Department of Agriculture, Agricultural Marketing Service — Miscellaneous Fruit and Vegetable Reports, Nos. 66 to 88, April and May 1941.

Questions

1. Does the use of daily price and quantity quotations reduce the danger of spurious correlation due to trend, seasonal and cyclical effects?

2. Refer to your scatter diagram and write a paragraph explaining whether or not a functional relationship appears to be present. Do you think that the relationship is linear or curvilinear? Explain.

3. Can you justify the type of function described in answering the above question (straight line or curvilinear relationship) on the basis of general economic theory as applied to competitive markets?

4. If you were to compute the regression line, which of the general types discussed in the text (arithmetic straight line, logarithmic straight line, second degree parabola, third degree parabola) do you think would be most satisfactory: (*a*) as a fit to the points, (*b*) from the standpoint of economic theory?

PROBLEM 88. NONSENSE CORRELATIONS

Unbridled participation in the sport of kings is likely to reduce those less affluent to penury. As a possible test of this generalization, we have collected data on states with pari-mutuel tax collections in 1947, and the number of commercial and industrial failures in 1947 for these states. These data are presented in Table 73.

TABLE 73

PARI-MUTUEL TAX COLLECTIONS AND INDUSTRIAL AND COMMERCIAL FAILURES, 1957

State	*Pari-Mutuel Tax Collections (Millions of $)*	*Industrial & Commercial Failures (No.)*
Arizona	.6	22
California	19.1	710
Florida	14.1	62
Illinois	6.1	214
Louisiana	.9	36
Maine	.3	22
Maryland	4.2	12
Massachusetts	10.7	237
Michigan	2.2	139
New Hampshire	3.2	7
New Jersey	7.7	157
New York	30.3	600
Ohio	.6	136
Oregon	.6	70
Rhode Island	4.8	35
Washington	.7	56
West Virginia	.2	18

Source: Pari-mutuel tax collections, *Facts and Figures in Government Finance 1948–1949*, The Tax Foundation, p. 92. *Industrial and Commercial Failures, 1947, by States*, Statistical Abstract, 1948, p. 474.

Instructions

1. Examine these data, graphically or otherwise, to determine whether they seem to be associated. A rough approximation rather than calculations with tests of significance will suffice.

Questions

1. Does any relationship which you may observe seem to be positive or negative?
2. To what do you attribute any correlation which is found?

PROBLEM 89. FUNCTIONAL RELATIONS

There are listed below several series which appear to be related. The problem is to discover the basic relationship which exists between the series in order to ascertain whether a functional relationship exists or whether other factors are responsible for the apparent correlation.

1. Passenger car registration and gasoline sales.
2. Cost of drilling a well per foot and depth of well.
3. Number of people passing a corner and site rent or value.
4. Temperature and fuel consumption.
5. Level of inventory of coal and price per ton.
6. Federal Reserve Board Index of Department Store Sales and National Income.
7. Cost of construction and rent in Chicago area.

PROBLEM 89. (Continued)

8. Miles of paved highway and motor oil sales.
9. Price of dressed steers and farm income.
10. Demand and price.
11. Supply and price.
12. Per capita federal debt and number of federal civil employees.
13. Number of telephones in a county and consumption of butter.
14. Federal Reserve Board Index of Industrial Production and Bureau of Labor Statistics Wholesale Price Index.
15. Employment and the Dow-Jones stock averages, New York Stock Exchange.
16. Gross national product and time, as measured in years, 1940, 1941, etc.

PROBLEM 90. REVIEW QUESTIONS ON CORRELATION

1. As a statistician for a business firm you are questioned regarding the significance of a coefficient of correlation, $r = -.60$. What further information, if any, would you require before coming to a decision?

2. If the coefficient of alienation in a correlation problem was determined as .60 and $b = -21.735$, what would be the value of the coefficient of correlation?

3. If in the regression equation the value of b was determined as zero, what would be the value of the coefficient of alienation? The coefficient of determination?

4. How is the sign of the coefficient of correlation determined?

5. *Rho*, the index of correlation, varies from zero to one, while r, the coefficient of correlation, varies from -1 to 0 and from 0 to $+1$. Why is this the case?

6. Is $r = .60$ significantly different from $r = 0$ when $N = 10$? When $N = 20$?
Is $r = .30$ significantly different from $r = 0$ when $N = 145$?

7. The coefficient of determination, r^2, plus the coefficient of non-determination, k^2, equals 1. Does $r + k$ ever equal 1? Under what circumstances?

8. If the value you computed for b in Problem 86 had been higher, would the correlation have been significantly greater?

9. If the standard deviation of the Y-series and the standard error of estimate of Y are identical, what would be the coefficient of correlation as computed by the method used in Problem 86? How would you explain this result?

SECTION XI

Appendix

Laboratory Exercises for Beginners with Computing Machines

Tables of Areas of the Normal Curve, of Square Roots, of Logarithms and Random Numbers

Instructions for the Use of a Slide Rule

Supplemental Problem 1

This problem is designed merely to give experience in the operation of the machines.

I. *Add* on the machines:

(*a*)	(*b*)
1678	72
10	891
792	5035
80761	627
9	48

(*c*) Check your additions by repeating in reverse order.

II. *Subtract* on the machines:

(*a*) 12,764.52 − 764.52 =
(*b*) 194,763.45 − 136,278.98 =
(*c*) 7849.65 − 9572.86 =

(*d*) Check your subtractions by adding the remainder to the subtrahend.

III. *Multiplication.* On the machines multiplication is by repeated addition. Thus: (12 × 23 = 276)

12
12
12
12
12
276

Multiply on the machines:

(*a*) 12 × 23 = 276
(*b*) 7026 × 43 =
(*c*) 39241 × 305 =

IV. *Division*, on the machines, is repeated subtraction.

(276 ÷ 12 = 23)
276
−12
15
−12
36
−12
24
−12
12
−12
0

Divide on the machines:

(*a*) 276 ÷ 12 =
(*b*) 9061 ÷ 85 =
(*c*) 46793 ÷ 779 =
(*d*) Check your division by multiplying the divisor by the quotient.
(*e*) Return to part III. Check the results of your multiplication by dividing the product by either the dividend or the divisor.

Supplemental Problem 2

This problem is intended to provide experience in the location of decimals, and computations involving fractions, ratios, division by multiplication, and square root.

Required Reading: Appendix A of text beginning on p. 683; Parts I, III, IV, V, and VI.

I. *Complete* as indicated.

(*a*) .673 × .27 =
(*b*) 67.3 × 2.7 =
(*c*) 6.73 × 27 =
(*d*) 673 × 7 =
(*e*) .673 ÷ .27 =
(*f*) 67.3 ÷ 2.7 =
(*g*) 6.73 ÷ .27 =

II. *Add*

(*a*) 7/8 plus 3/24 =
(*b*) 8/16 plus 1¼ =
(*c*) ⅖ plus ⅓ =

III. *Multiply*

(*a*) 7/8 × 3/24 =
(*b*) ⅖ × ⅓ =

IV. Reduce several of the fractions under II above to decimal fractions, for example:

(*a*) 7/8 = .875
(*b*) 3/24 =
(*c*) ⅖ =

V. *Percentage*

(*a*) What percentage is 468 of 234? _____ By what per cent does 468 exceed 234? _____

(*b*) What percentage is .468 of .234? _____

(*c*) What per cent is 52,789 of 47,678? _____ What is per cent by which 52,789 exceeds 47,678? _____

(*d*) We have the following index numbers for each of three years:

1939	89
1946	168
1948	148

(1) What per cent is 1946 value of 1939 value? _____ What is the per cent of change? _____

(2) What per cent is the 1948 value of the 1946 value? _____ What is the per cent of change? _____

(*e*) We have the following record of sales:

	1940	1947	Percentages 1940	Percentages 1947
Building hardware	$65,000	$115,000	_____	_____
Electrical appliances	86,000	210,000	_____	_____
Power tools	21,000	42,000	_____	_____
Hand tools	7,000	10,000	_____	_____
Totals				

(1) Compute the per cent which each category of sales is of total sales each year.
(2) Check your percentages to see that they total 100.
(3) Are the percentages for building hardware, power tools, and hand tools higher in 1940 than in 1947? _____
(4) Are the actual sales in the three categories mentioned in (*c*) higher than in 1947? _____
(5) What was the percentage increase in total sales, 1940–1947? _____
(6) What was the percentage increase in each category of sales 1940–1947? _____

VI. *Division by multiplication*

We have the following numbers, all of which are to be divided by the number 8. (See part VI of Appendix A)

$648 \div 8 =$
$536 \div 8 =$
$872 \div 8 =$

Because, for example, $\frac{648}{8} = 648 \times \frac{1}{8}$ we may take the reciprocal of 8 and multiply each of the numbers by it.

$648 \times$ _____ $=$
$536 \times$ _____ $=$
$872 \times$ _____ $=$

Do the two methods give identical results?

VII. *Square root.* It is required that all students know how to compute the square root. (Part V of Appendix A)

(*a*) Look up the square root of 9.61 in Table III, Appendix E. _____
(*b*) Look up the square root of 96.10 in Table III, Appendix E. _____
(*c*) Extract the square root of the two numbers given in (*a*) and (*b*) by hand.

TABLE I

AREAS OF THE NORMAL CURVE FROM MEAN TO INDICATED DEVIATION FROM MEAN

x/σ	00	01	02	03	04	05	06	07	08	09
0.0	00000	00399	00798	01197	01595	01994	02392	02790	03188	03586
0.1	03983	04380	04776	05172	05567	05962	06356	06749	07142	07535
0.2	07926	08317	08706	09095	09483	09871	10257	10642	11026	11409
0.3	11791	12172	12552	12930	13307	13683	14058	14431	14803	15173
0.4	15542	15910	16276	16640	17003	17364	17724	18082	18439	18793
0.5	19146	19497	19847	20194	20540	20884	21226	21566	21904	22240
0.6	22575	22907	23237	23565	23891	24215	24537	24857	25175	25490
0.7	25804	26115	26424	26730	27035	27337	27637	27935	28230	28524
0.8	28814	29103	29389	29673	29955	30234	30511	30785	31057	31327
0.9	31594	31859	32121	32381	32639	32894	33147	33398	33646	33891
1.0	34134	34375	34614	34850	35083	35314	35543	35769	35993	36214
1.1	36433	36650	36864	37076	37286	37493	37693	37900	38100	38298
1.2	38493	38686	38877	39065	39251	39435	39617	39796	39973	40147
1.3	40320	40490	40658	40824	40988	41149	41309	41466	41621	41774
1.4	41924	42073	42220	42364	42507	42647	42786	42922	43056	43189
1.5	43319	43448	43574	43699	43822	43943	44062	44179	44295	44408
1.6	44520	44630	44738	44845	44950	45053	45154	45254	45352	45449
1.7	45543	45637	45728	45818	45907	45994	46080	46164	46246	46327
1.8	46407	46485	46562	46638	46712	46784	46856	46926	46995	47062
1.9	47128	47193	47257	47320	47381	47441	47500	47558	47615	47670
2.0	47725	47778	47831	47882	47932	47982	48030	48077	48124	48169
2.1	48214	48257	48300	48341	48382	48422	48461	48500	48537	48574
2.2	48610	48645	48679	48713	48745	48778	48809	48840	48870	48899
2.3	48928	48956	48983	49010	49036	49061	49086	49111	49134	49158
2.4	49180	49202	49224	49245	49266	49286	49305	49324	49343	49361
2.5	49397	49396	49413	49430	49446	49461	49477	49492	49506	49520
2.6	49534	49547	49560	49573	49585	49598	49609	49621	49632	49643
2.7	49653	49664	49674	49683	49693	49702	49711	49720	49728	49736
2.8	49744	49752	49760	49767	49774	49781	49788	49795	49801	49807
2.9	49813	49819	49825	49831	49836	49841	49846	49851	49856	49861
3.0	49865	49869	49874	49878	49882	49886	49889	49893	49897	49900
3.1	49903	49906	49910	49913	49916	49918	49921	49924	49926	49929
3.2	49931	49934	49936	49938	49940	49942	49944	49946	49948	49950
3.3	49952	49953	49955	49957	49958	49960	49961	49962	49964	49965
3.4	49966	49968	49969	49970	49971	49972	49973	49974	49975	49976
3.5	49977	49978	49978	49979	49980	49981	49981	49982	49983	49983
3.6	49984	49985	49985	49986	49986	49987	49987	49988	49988	49989
3.7	49989	49909	49909	49909	49991	49991	49992	49992	49992	49992
3.8	49993	49993	49993	49994	49994	49994	49994	49995	49995	49995
3.9	49994	49995	49996	49996	49996	49996	49996	49996	49997	49997
4.0	49997	49997	49997	49997	49997	49997	49998	49998	49998	49998

TABLE II

Square Root Table

N	$\sqrt{N}$	$\sqrt{10N}$	N	$\sqrt{N}$	$\sqrt{10N}$
1	1.0000	3.1623	50	7.0711	22.3607
2	1.4142	4.4721	51	7.1414	22.5382
3	1.7320	5.4772	52	7.2111	22.8035
4	2.0000	6.3246	53	7.2801	23.0217
			54	7.3485	23.2379
5	2.2361	7.0711			
6	2.4495	7.7460	55	7.4162	23.4521
7	2.6458	8.3667	56	7.4833	23.6643
8	2.8284	8.9443	57	7.5498	23.8747
9	3.0000	9.8468	58	7.6158	24.0832
			59	7.6811	24.2899
10	3.1623	10.0000			
11	3.3166	10.4881	60	7.7459	24.4940
12	3.4641	10.9544	61	7.8103	24.6982
13	3.6055	11.4018	62	7.8740	24.8998
14	3.7417	11.8322	63	7.9372	25.0998
			64	8.0000	25.2982
15	3.8730	12.2475			
16	4.0000	12.6491	65	8.0625	25.4951
17	4.1231	13.0384	66	8.1240	25.6905
18	4.2426	13.4164	67	8.1853	25.8843
19	4.3589	13.7841	68	8.2462	26.0768
			69	8.3066	26.2678
20	4.4721	14.1421			
21	4.5826	14.4914	70	8.3666	26.4575
22	4.6904	14.8324	71	8.4261	26.6458
23	4.7958	15.1658	72	8.4853	26.8328
24	4.8989	15.4919	73	8.5440	27.0185
			74	8.6023	27.2029
25	5.0000	15.8114			
26	5.0990	16.1245	75	8.6602	27.3861
27	5.1962	16.4317	76	8.7178	27.5681
28	5.2915	16.7332	77	8.7749	27.7489
29	5.3851	17.0294	78	8.8317	27.9285
			79	8.8882	28.1069
30	5.4772	17.3205			
31	5.5677	17.6068	80	8.9443	28.2843
32	5.6569	17.8885	81	9.0000	28.4605
33	5.7446	18.1659	82	9.0554	28.6356
34	5.8310	18.4391	83	9.1104	28.8097
			84	9.1651	28.9827
35	5.9161	17.7083			
36	6.0000	18.9737	85	9.2195	29.1548
37	6.0828	19.2354	86	9.2736	29.3258
38	6.1644	19.4936	87	9.3274	29.4958
39	6.2449	19.7484	88	9.3808	29.6648
			89	9.4339	29.8329
40	6.3245	20.0000			
41	6.4031	20.2485	90	9.4868	30.0000
42	6.4807	20.4939	91	9.5394	30.1662
43	6.5574	20.7364	92	9.5917	30.3315
44	6.6332	20.9761	93	9.6436	30.4959
			94	9.6954	30.6594
45	6.7082	21.2132			
46	6.7823	21.4476	95	9.7467	30.8221
47	6.8556	21.6795	96	9.7980	30.9839
48	6.9282	21.9089	97	9.8489	31.1448
49	7.0000	22.3607	98	9.8995	31.3049
			99	9.9499	31.4643
			100	10.0000	31.6228

TABLE III

FOUR PLACE LOGARITHM

No.	0	1	2	3	4	5	6	7	8	9	Proportional Parts					
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374		38	36	34	32	30
11	0414	0453	0492	0531	0569	0607	0654	0682	0719	0755	.1	4	4	3	3	3
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	.2	8	7	7	7	6
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	.3	11	11	10	10	9
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	.4	15	14	14	13	12
											.5	19	18	17	16	15
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	.6	23	22	20	19	18
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	.7	27	25	24	22	21
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	.8	30	29	27	26	24
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	.9	34	32	31	29	27
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989						
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201		28	26	24	22	20
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	.1	3	3	2	2	2
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	.2	6	5	5	4	4
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	.3	8	8	7	7	6
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	.4	11	10	10	9	8
											.5	14	13	12	11	10
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	.6	17	16	14	13	12
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	.7	20	18	17	15	14
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	.8	22	21	19	18	16
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	.9	25	23	22	20	18
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757						
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900		18	16	15	14	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	.1	2	2	2	1	1
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	.2	4	3	3	3	3
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	.3	5	5	5	4	4
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	.4	7	6	6	6	5
											.5	9	8	8	7	7
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	.6	11	10	9	8	8
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	.7	13	11	11	10	9
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	.8	14	13	12	11	10
38	5798	5809	5821	4832	5843	5855	5866	5877	5888	5899	.9	16	14	14	13	12
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010						
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117		12	11	10	9	8
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	.1	1	1	1	1	1
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	.2	2	2	2	2	2
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	.3	4	3	3	3	2
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	.4	5	4	4	4	3
											.5	6	6	5	5	4
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	.6	7	7	6	5	5
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	.7	8	8	7	6	6
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	.8	10	9	8	7	6
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	.9	11	10	9	8	7
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981						
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067						
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152						
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235						
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316						
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396						

TABLE III (Continued)

Four Place Logarithm

No.	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	*8000*	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9166	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9451	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9947	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

Proportional Parts					
9	*8*	*7*	*6*	*5*	*4*
1	1	1	1	1	0
2	2	1	1	1	1
3	2	2	2	2	1
4	3	3	2	2	2
5	4	4	3	3	2
5	5	4	4	3	2
6	6	5	4	4	3
7	6	6	5	4	3
8	7	6	5	5	4

TABLE IV

RANDOM NUMBERS

Line \ Col.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1051	02946	96520	61881	56247	17623	47441	27821	91845	15714	47300	41511	44365	99898	56087
1052	85697	62000	87957	07258	45054	58410	92081	97624	03586	88294	64116	94726	31820	74067
1053	26734	68426	58067	23123	73700	58730	06111	64486	82045	25637	90031	43964	39009	79954
1054	47829	32355	95941	72169	58374	03905	06865	95353	23101	41360	76055	74426	59237	70699
1055	76603	99339	40571	41186	04981	17531	97372	39558	69068	07420	84874	90220	46055	42857
1056	47526	26522	11045	83565	45639	02485	43905	01823	11997	55770	04894	13660	18373	20740
1057	70100	85732	19741	92951	98832	38188	24080	24519	62748	54356	00260	10051	82738	90022
1058	86819	50200	50889	06493	66638	03619	90906	95370	43420	00980	29684	66876	70294	47371
1059	41614	30074	23403	03656	77580	87772	86877	57085	88032	58290	15614	62605	45459	69371
1060	17930	26194	53836	53692	67125	98175	00912	11246	60393	34510	50509	62330	70152	20836
1061	24649	31845	25736	75231	83808	98997	71829	99430	99629	44443	24165	50988	92208	49492
1062	79899	34061	54308	59358	56462	58166	97302	86828	08415	18111	94115	47281	41826	38774
1063	76801	49594	81002	30397	52728	15101	72070	33706	83438	02756	46441	21655	30827	35016
1064	62567	08480	61873	63162	44873	35302	04511	38088	96236	03668	94183	03773	63502	40017
1065	49723	15275	09399	11211	67352	41526	23497	75440	46317	38123	75503	45577	45525	38725
1066	42658	70183	89417	57676	35370	14915	16569	54945	35358	73388	28224	21814	02558	40824
1067	65080	35569	79392	14937	06081	74957	87787	68849	78085	63685	57537	39437	41895	16115
1068	02906	38119	72407	71427	58478	99297	43519	62410	12242	90647	87380	53942	08294	91798
1069	75153	86376	63852	60557	21211	77299	74967	99038	57612	97072	51406	96836	26372	52131
1070	14192	49525	78844	13664	98964	64425	33536	15079	00091	87240	54274	86634	86702	23050
1071	32059	11548	86264	74406	81496	23996	56872	71401	59284	42455	71966	48720	99905	13769
1072	81716	80301	96704	57214	71361	41989	92589	69788	20721	82236	18597	40883	61703	57145
1073	43315	50483	02950	09611	36341	20326	37489	34626	81690	52379	75898	20851	36483	90898
1074	27510	10769	09921	46721	34183	22856	18724	60422	94578	44051	01563	48610	67094	21667
1075	81782	04769	36716	82519	98272	13969	12429	03093	89469	41177	17115	79207	64792	38623
1076	19975	48346	91029	78902	75689	70722	88553	83300	84330	95346	45931	28180	98118	66069
1077	98356	76855	18769	52843	64204	95212	31320	03783	26906	84265	17840	50625	30699	53192
1078	29708	17814	31556	68610	16574	42305	56300	84227	59147	18934	36923	83101	14609	61318
1079	88014	27583	78167	25057	93552	74363	30951	41367	81193	86710	51226	27067	33267	60753
1080	94491	19238	17396	10592	48907	79840	34607	62668	29680	14786	17902	83847	35528	34860
1081	56957	05072	53948	07850	42569	82391	20435	79306	91119	02988	58792	03362	84272	31787
1082	50915	31924	80621	17495	81618	15125	48087	01250	59915	47784	06495	30072	17575	59835
1083	49631	93771	80200	84622	31413	33756	15218	81976	79409	94895	23730	66665	06133	52448
1084	99683	58162	45516	39761	77600	15175	67415	88801	77378	91513	89678	07672	57062	91729
1085	86017	20264	94618	85979	42009	78616	45210	73186	03869	02881	23638	41792	11325	91653
1086	77339	64605	82583	85011	02955	84348	46436	77911	50191	19372	16950	39034	15032	84081
1087	61714	57933	37342	26000	93611	93346	71212	24405	18846	87224	59101	45552	54241	36523
1088	15232	48027	15832	62924	11509	95853	02747	61889	08567	76588	07718	32505	37265	26390
1089	41447	34275	10779	83515	63899	30932	90572	98971	66287	38099	12635	87644	33457	43804
1090	23244	43524	16382	36340	73581	76780	03842	64009	95845	60495	72208	80379	99160	55477
1091	53460	83542	25224	70378	49604	14809	12317	78062	38320	21331	50681	10089	52276	93264
1092	53442	16897	61578	05032	81825	76822	87170	77235	43138	65496	50955	38017	12609	16724
1093	55548	19096	04130	23104	60534	44842	16954	99466	37192	28518	63412	54607	62493	60940
1094	18185	69329	02340	63111	41788	74409	76177	55519	16443	60181	32185	54956	17203	62876
1095	02372	45690	38595	23121	73818	74454	02371	94693	72065	88784	28349	89472	24577	29429
1096	51715	35492	61371	87132	81585	55439	98095	55578	32831	61333	70640	95003	76601	18201
1097	24717	16786	42786	86985	21858	39489	39251	70450	32283	74130	21070	45390	65047	47577
1098	78022	32604	87259	93708	99438	68184	62119	20229	90328	25634	90890	65331	25416	67162
1099	35995	08275	62405	43313	03249	74135	43003	63132	00852	40147	53963	13009	85861	81690
1100	29192	86922	31908	42703	59638	31226	89860	45191	13098	00618	76390	86918	54584	59212

TABLE IV (Continued)

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1102	21791	24764	53674	30093	45134	84073	24316	59727	95068	88513	63804	44547	19920	37719
1103	63501	05040	71881	17759	91881	69614	11891	27729	60387	80028	56757	90152	85907	27714
1104	07149	69285	55481	21889	67061	06631	27033	80489	97300	74647	95818	13557	92510	22063
1105	59443	98962	74778	96920	65620	36794	21088	06901	66167	48737	32171	41508	64576	30029
1106	39099	58021	28486	43052	99201	44400	12566	40060	00110	92074	99887	52372	31852	04306
1107	78176	58913	22638	69769	21102	72292	79494	53854	21234	84810	79995	74837	77566	54222
1108	11851	09065	98033	02752	58232	56504	22187	40028	80458	54883	37743	03336	89554	49006
1109	37515	25668	55785	66463	52758	67588	89891	47358	60805	96396	96460	70424	84511	69709
1110	45324	00016	46818	04873	75360	87519	65700	83430	99497	68213	98839	10962	61685	14741
1111	39338	62200	12200	28056	51447	92050	36908	42699	03714	77249	72501	85634	60027	90534
1112	48107	22333	37919	63359	60491	12582	23078	39200	81789	94129	06981	17417	36781	88738
1113	34582	52485	66816	98008	48509	23046	64998	75587	39586	73181	69189	73080	90698	22391
1114	11726	87723	17350	66656	25153	29295	30539	68093	19700	72068	46802	15591	10475	59195
1115	62681	84947	71441	44095	06799	51965	96630	34517	86340	70145	32256	99884	21945	24901
1116	19024	78592	25380	36560	49782	92033	98834	37738	21520	97739	66229	01420	45984	61386
1117	03788	52694	48458	93740	39643	73505	90863	10053	90470	24877	02811	03948	92737	21250
1118	65679	13628	89003	91555	21634	65067	85299	89657	64650	31564	74380	80779	73446	32001
1119	88772	38763	90586	17112	30425	84155	68208	53980	54149	02083	39615	17901	05289	75823
1120	63199	69880	78978	20867	57997	58471	23808	18518	39160	39984	56686	62181	29719	15680
1121	57013	40387	28838	70866	60528	03192	37728	58364	46511	78976	93796	29389	77418	00645
1122	81991	17816	21981	56841	77089	90232	98880	68900	04347	79116	98308	90018	68662	84028
1123	95637	56711	21483	23419	57836	94890	75997	95806	69840	96440	38158	54219	95388	69676
1124	37603	31906	85392	61483	94920	28886	87488	62857	86680	81458	19190	13618	86274	91991
1125	14316	23028	93147	58643	26761	34390	04023	04460	36456	36566	78618	99437	86280	47753
1126	48133	15695	19836	69538	92344	86830	08195	57578	23807	64850	23740	22393	84405	46730
1127	80104	03086	43808	65429	61701	99787	33860	86452	32956	55928	88298	50320	11042	33439
1128	56644	71608	78728	00126	37806	18748	72277	86832	94521	40759	25945	42118	60792	71338
1129	82240	27798	82943	51028	15020	12888	80359	39004	84963	95302	51079	16914	47121	00530
1130	36691	35957	30564	38452	20611	38423	01639	33168	43487	49113	73030	88466	26159	72848
1131	80883	38042	40889	86235	89617	02462	13285	26584	97124	01278	01371	82210	37741	07007
1132	86028	90782	03808	87781	13877	66987	62561	19821	92836	83691	09632	66384	95619	48647
1133	78796	20010	73633	78747	43470	61915	89760	68086	64966	47841	76044	29599	81688	31699
1134	01895	76818	47221	62383	17760	23087	57890	48237	86058	29897	32146	69086	01147	48562
1135	38336	16181	60770	27481	61366	83454	59127	46917	96807	28423	69102	42569	96585	01468
1136	60159	45585	84667	76841	25040	64123	62982	01259	16079	73837	30734	07512	89405	24384
1137	13620	69865	19833	25094	71527	27540	31297	49742	86668	81255	94208	72963	85699	65130
1138	61129	88150	46133	85056	87098	74354	15694	29175	34091	05041	83851	76151	01422	87630
1139	10683	69313	42008	68280	07110	79227	68750	77140	04429	01015	64858	92715	38553	31158
1140	90057	80871	40180	59371	15844	50034	38356	98986	68617	83627	87184	23160	98540	01607
1141	90903	02994	19299	37195	96116	59300	05753	78285	66608	94618	07285	24214	83835	46617
1142	76759	88718	84511	41008	18443	90015	37661	45306	70387	87002	82771	52847	96796	51101
1143	48876	06965	87207	01521	72643	54376	51819	67447	15548	81039	21389	18600	90951	76959
1144	34053	46074	09909	93541	17568	78648	21245	51523	96825	58814	06923	89805	08217	36825
1145	02707	58000	48515	92480	31701	28374	57525	94047	14995	32368	67478	93903	28352	18313
1146	86946	26519	81177	17594	55005	80011	76723	49927	74005	74816	31431	51417	90446	34498
1147	08609	40279	37084	17744	24046	86794	89036	38806	98386	72575	78105	13100	01522	54198
1148	83698	96268	38476	24281	76576	66333	12180	45407	00564	41920	10188	34969	18223	84251
1149	47164	96557	14200	88122	98488	47335	24463	66676	90028	42198	78152	37817	99951	01722
1150	16308	15528	53067	84067	79343	42662	56819	40187	40001	48553	35741	03662	26449	98922

TABLE IV (Continued)

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1152	73750	68343	40727	81203	91727	06463	12248	57567	88750	57207	74771	63286	52041	37502
1153	64163	22132	22896	14305	45642	71580	21558	66457	87216	14927	02015	01218	11727	19528
1154	88445	85544	23627	79176	32502	34568	78777	35179	90882	48719	56575	54346	32740	80844
1155	94180	71108	19121	11958	33308	75930	24865	17426	46053	16954	09323	46786	52842	80029
1156	11433	12220	36719	35435	77727	78493	94580	70091	88108	68433	93301	41968	22860	06589
1157	61638	68801	49856	21739	95701	11180	73936	17628	12311	68680	73549	42393	32358	14055
1158	59908	68103	36855	19127	56637	06383	83182	76927	03134	60092	04961	22440	28532	75148
1159	41018	69556	06402	03436	89803	95604	55735	66978	70815	20734	24410	38543	81884	30318
1160	85095	01581	92299	06166	71078	31823	64316	95567	42508	63549	67909	19847	75922	42309
1161	59705	78103	66740	41743	69177	59277	47629	63874	95777	27606	55163	66908	77090	73600
1162	75094	55208	77905	20705	32408	16630	01242	63119	24272	40966	59365	45037	94748	45130
1163	78425	68672	79455	94334	23292	90422	75540	15843	03536	78929	27952	51325	38999	71222
1164	89088	86918	20787	05691	97309	96673	16389	51437	67463	62091	04531	68337	14487	03803
1165	83345	95889	39333	86027	24680	51909	97230	58136	00901	36504	79092	24556	64013	55752
1166	62896	00342	66647	57096	84913	67895	18804	17691	79985	02249	05018	93491	90608	78421
1167	96498	38270	80532	54307	07885	38892	50990	96766	10391	47494	61462	26236	50785	70081
1168	30974	47335	04918	42974	19294	72581	77377	04652	32773	77247	07213	68838	84135	88987
1169	57901	06163	99162	53285	27341	02507	41858	08436	95117	59801	64130	02595	05587	07478
1170	88494	80633	47765	53996	57058	04222	54488	12019	19961	98141	35105	55638	06283	21760
1171	34883	00045	89682	86664	92195	42593	56488	35402	03458	41818	29466	74119	02268	45858
1172	24373	46438	28935	63903	14722	10715	58795	42800	72900	58210	69550	27267	08158	59599
1173	16828	79262	23678	05509	23733	95318	77730	87614	75047	86064	83464	25290	76619	95392
1174	33723	27646	92335	87136	88062	21506	01750	71326	79990	72900	72778	42614	61003	76848
1175	01542	75066	73921	97188	31250	41996	31680	41783	58423	77216	06815	12302	03074	86293
1176	00100	12787	74100	95536	42359	01761	28842	71562	50387	85776	89783	95928	48924	40650
1177	82697	03389	19303	21646	22532	81701	03425	28914	68714	07516	21639	51991	86496	31881
1178	28137	17549	22698	72955	59849	02370	02784	13711	16466	51514	85699	30894	56668	95708
1179	02248	21570	33796	83789	72981	96423	68791	91684	63140	27870	37517	54420	33404	15825
1180	56175	82515	23348	42207	87644	57353	90349	16448	19161	99292	42009	31469	78108	59662
1181	80020	21622	67659	07878	17586	65524	20162	04712	31781	90129	72060	41170	23012	05969
1182	20271	23094	48372	77621	32889	19595	66500	28064	72134	85573	13428	55943	09667	75600
1183	38734	98044	02658	90698	72563	15076	23780	52815	24558	63962	95247	66493	79512	06464
1184	48183	24263	49297	32923	94406	63865	44336	27224	15045	59875	89505	20869	82810	55930
1185	48163	34158	03177	51696	57795	31725	14403	29856	64364	74978	12011	03976	55525	91112
1186	45658	15024	66664	18730	40671	92727	68626	81631	62377	33816	95294	34344	17329	58211
1187	71128	15524	55666	14763	13729	51708	54104	81331	03302	62068	55783	19096	59544	63944
1188	19041	42899	49464	93965	14960	88896	72784	82054	54161	77417	15678	87019	56787	91129
1189	32672	67506	93040	94527	31556	80163	80203	90928	95719	97206	69445	26889	57665	54865
1190	15823	48310	04391	15521	79255	69253	60254	01653	79162	36475	30155	35981	45176	75237
1191	82810	18981	62581	31642	42693	78972	60322	90462	04911	81323	80654	01210	18122	21060
1192	74772	80840	05816	29023	67410	12916	87933	78840	35477	28422	06146	02961	58363	15967
1193	52931	38199	85632	23761	99084	48028	07184	41635	90549	85312	04208	21329	92628	28400
1194	95395	87644	09722	99251	97129	70847	91864	08549	43675	39935	29510	57670	10593	45735
1195	76695	33451	57139	90612	11918	90871	60965	23555	29024	09147	34486	53759	75675	19516
1196	83560	50374	04410	57272	36705	51302	93147	29479	09307	32397	16313	31653	74323	03129
1197	28355	62002	85994	35807	84810	14186	51153	78998	40817	17031	97968	26048	86716	93709
1198	84684	54861	41330	66808	65231	14168	45193	27156	63562	37368	57576	03175	05539	96923
1199	21135	92001	43896	55887	35319	03793	60344	95970	86742	16043	87224	01591	77173	26690
1200	24236	01536	43997	41294	45551	46877	58631	82654	51804	67284	36564	70762	40619	86658

TABLE IV (Continued)

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1202	93024	46250	20788	59964	81609	87694	40186	77855	91912	90065	25480	90939	62906	77020
1203	00974	03860	69142	00782	53189	44463	38322	68645	04153	73446	72483	96092	19099	27844
1204	85980	89568	86897	28819	93015	40804	45389	22679	62921	82470	41981	56496	43516	53703
1205	21926	84853	40364	14457	53939	41653	71979	81588	76284	98952	93527	00755	40680	16398
1206	96433	60242	22116	90230	87423	30668	35055	11813	81689	26586	80483	56892	72824	36970
1207	75535	03894	97949	64908	05890	70809	62170	04536	22023	23225	32445	48256	60669	35184
1208	15524	33007	98091	29915	57545	46078	03871	20516	23655	25994	45375	33270	01238	81788
1209	24189	85050	98663	38560	52553	81991	75859	52012	84186	14314	02003	12032	34204	30000
1210	83846	38182	07792	45967	41951	49621	82062	95925	48928	31720	92157	85869	51119	04031
1211	46739	32153	32959	10736	18932	13905	88740	19460	37435	43506	71987	72845	61462	76830
1212	41769	93463	66194	41318	13775	85724	36153	61195	49203	30323	68780	50706	07190	36282
1213	51182	90259	87138	92471	60295	10007	83366	18389	51091	48781	96234	72536	13147	93207
1214	47466	94179	64271	72693	41484	88010	67198	38679	56009	38993	19716	75096	35421	78977
1215	07835	74209	01787	22698	83567	81719	66085	75038	77919	56063	24978	68642	40122	44761
1216	56347	54336	56090	23776	75111	25829	40173	09882	22120	51197	37782	33145	60469	86554
1217	06888	32638	76714	83789	06638	09669	17406	40741	81363	93117	50682	40594	74399	33035
1218	35128	85695	59201	85555	41254	00703	73159	42562	58915	25518	10960	33930	27294	90639
1219	39612	93090	62253	15598	25911	65423	48300	94823	64886	92193	30423	35724	50001	70181
1220	78265	13257	29335	28987	55262	11962	21679	49924	29840	58689	81127	26594	22253	00935
1221	59329	90504	14855	84017	46761	29773	39746	37306	65238	60160	55801	48715	07263	07887
1222	09419	16715	89918	73266	36691	81481	40067	08081	14640	82156	24494	93889	93302	81493
1223	70316	73177	86655	51109	23593	43166	22575	19703	68559	74737	04236	49146	05485	66292
1224	42520	64431	56085	94988	65818	42961	92714	18925	58377	65806	60920	28335	08675	76171
1225	77795	37676	38164	39446	70684	39200	60688	02706	96807	55638	22967	06037	24814	36489
1226	96827	83921	94896	83396	27299	39388	45625	87659	42207	87885	23017	39734	60416	85488
1227	30509	37575	34107	61451	57192	28198	40088	01627	45649	08101	87770	53605	47531	54647
1228	48141	52622	68769	88507	15873	77512	43892	06278	68781	51205	69464	86145	61598	31280
1229	76775	13782	08331	26338	75128	26329	00011	82457	05784	18745	83927	50635	71277	38591
1230	80039	80106	61471	85010	57652	86208	51828	36666	67629	93189	82485	47927	17545	45710
1231	50113	18798	40463	27292	63786	89526	00611	38869	57250	46947	03270	49364	77561	51844
1232	70366	77355	78738	22944	28371	13278	75571	67914	11870	91882	72954	77650	91559	26554
1233	06521	04544	32907	33270	11277	03409	49440	15467	86398	11106	25506	18874	76940	02591
1234	44674	13886	70818	44288	98331	86481	28733	92684	50536	81567	22092	98683	67957	69799
1235	28082	15034	60185	14396	71652	17739	20900	54226	16312	91740	64103	04591	11191	97244
1236	74421	99841	80044	34782	16680	04551	63025	15146	24583	49797	60371	78626	03758	99361
1237	54667	63363	58192	26084	31078	11497	40343	65865	57238	74354	66207	57910	14211	25688
1238	17153	99397	13760	73178	37357	45190	06190	17062	81561	89720	37380	98109	90904	54969
1239	71721	67421	84805	50388	77723	86202	58475	75317	05427	79345	11986	07102	54013	40134
1240	78150	57466	19071	89888	07223	96310	80678	99686	00442	60341	81608	66515	98409	23116
1241	22575	85710	42407	09281	07074	02259	47903	43970	59180	18448	47771	82050	49732	66456
1242	51757	06816	15999	06704	12411	46373	08176	51644	16545	41250	80827	02713	45634	35497
1243	76793	84715	95520	96722	44927	14891	42580	05714	56252	75024	66387	90298	18461	58897
1244	64706	43607	95687	84579	65005	50318	98010	43800	47285	86501	22830	42132	51268	61228
1245	01619	24617	23748	43240	20646	66685	06769	14617	25884	06363	15587	68373	09650	53757
1246	66396	79480	08417	07971	42258	66401	89016	00576	56554	92936	43549	70432	96456	13088
1247	88247	92465	86970	27056	75111	35954	04908	39278	35048	22782	57676	02777	85138	13520
1248	34573	11739	68894	31311	68741	06781	16665	56810	54709	71875	86136	52822	03607	16646
1249	16436	72091	47835	13637	03540	50764	33141	61687	44597	55932	70676	38083	66787	93793
1250	12577	67820	43932	31786	09878	30754	35826	55219	49221	51126	11788	45411	53835	37783

Source: "Table of 150,000 Random Decimal Digits," Interstate Commerce Commission, Bureau of Transport Economics and Statistics, May 19, 1949, Washington, D. C. Four pages reproduced by permission.

On the Use of a Slide Rule

The slide rule is an extremely handy tool for the analyst for obtaining quotients, ratios, products, roots and squares, where required accuracy is not greater than three significant numbers. Values beyond this point are enough subject to human and mechanical error to impair the validity of slide rule readings. However, if the limited accuracy of the slide rule is recognized and use appropriately restricted, the few moments required to master the technique will be well repaid. In using a slide rule, it is necessary to determine the proper number of decimal places by inspection

Division

To divide, place the slide indicator so that the hairline is directly over the dividend on Scale D and adjust the sliding portion of the rule until the divisor on Scale C is directly over the hairline. Read the answer on the D Scale under the 1 (or 10) over the C Scale.

For example: $18 \div 3$

Place 3 on the C Scale over 18 on D Scale and read value on D under 10 on C.

$6 \div 4$

Place 4 on the C Scale over 6 on D Scale and read value on D under 1 on C.

Multiplication

Multiplication is the opposite of division. To multiply, place the 1 (or 10) on the C Scale over one of the numbers to be multiplied on the D and read the answer on the D under the other multiplier on the C.

For example: 3×6

Place the 10 on the C Scale over the 3 on the D Scale and read the answer on the D under 6 on the C.

4×1.5

Place the 1 on the C Scale over the 4 on the D Scale and read the answer on the D under 1.5 on the C.

Square Root

To obtain the square root of a number, place the slide indicator over the number on the A Scale and read the square root under the hairline on the D Scale. (The square of a number is obtained by the opposite process.)

For example: $\sqrt{9}$

Place hairline over 9 on A Scale and read value, 3 on D Scale.

$\sqrt{81}$

Place hairline over 81 on A Scale and read value, 9 on D Scale.

SECTION XII

Work Materials